Sri Adichunchanagiri Shikshana Trust (R)

SJB Institute of Technology

(Affiliated to Visvesvaraya Technological University, Belagavi& Approved by AICTE, New Delhi.)

Department of Electrical & Electronics Engineering

Power System Analysis – 2 [18EE71]

Module-1

Network Topology

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Lecture 1

Discussion on Syllabus

Module 1: Network Topology

Module 2 & 3: Load Flow Studies

Module 4: Economic Operation of Power System & Unit Commitment

Module 5: Symmetrical Fault Analysis & Power System Stability

Syllabus

Module 1: Network Topology:

Introduction and basic definitions of Elementary graph theory Tree, cut-set, loop analysis. Formation of Incidence Matrices. Primitive network- Impedance form and admittance form, Formation of Y Bus by Singular Transformation. Y_{bus} by Inspection Method. Illustrative examples.

Basics

- Power
- Bus
- Node
- Mesh
- Loop
- Linear Network
- Non-linear Network
- Impedance
- Admittance

Introduction

- The solution of a given linear network problem requires the formation of a set of equations describing the response of the network.
- The mathematical model so derived, must describe the characteristics of the individual network components, as well as the relationship which governs the interconnection of the individual components.
- In the bus frame of reference the variables are the node voltages and node currents.

Introduction Continued..

- The independent variables in any reference frame can be either currents or voltages. Correspondingly, the coefficient matrix relating the dependent variables and the independent variables will be either an impedance or admittance matrix.
- The formulation of the appropriate relationships between the independent and dependent variables is an integral part of a digital computer program for the solution of power system problems.
- The formulation of the network equations in different frames of reference requires the knowledge of graph theory.

MCQ’s

1. The power system analysis is essential for

a)Planning the operation

b)Improvement and expansion of power system

c)Both a and b

d)None of the above

Lecture 2

Elementary Graph Theory

- The geometrical interconnection of the various branches of a network is called the network topology.
- The connection of the network topology, shown by replacing all its elements by lines is called a graph.
- A linear graph consists of a set of objects called nodes and another set called elements such that each element is identified with an ordered pair of nodes.
- An element is defined as any line segment of the graph irrespective of the characteristics of the components involved.
- A graph in which a direction is assigned to each element is called an oriented graph or a directed graph.

- The ground node is taken as the reference node. In electric networks the convention is to use associated directions for the voltage drops. This means the voltage drop in a branch is taken to be in the direction of the current through the branch. Hence, we need not mark the voltage polarities in the oriented graph.

Connected Graph: This is a graph where at least one path (disregarding orientation) exists between any two nodes of the graph. A representative power system and its oriented graph are as shown in Fig 1.1, with:

e = number of elements = 6

n = number of nodes = 4

b = number of branches = n-1 = 3

l = number of links = e-b = 3

Tree = T(1,2,3) and

Co-tree = T(4,5,6)

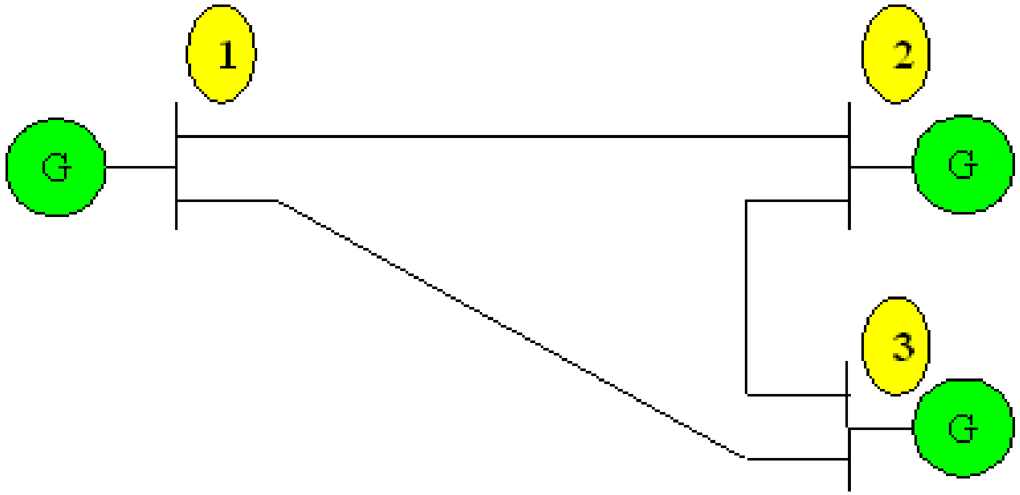
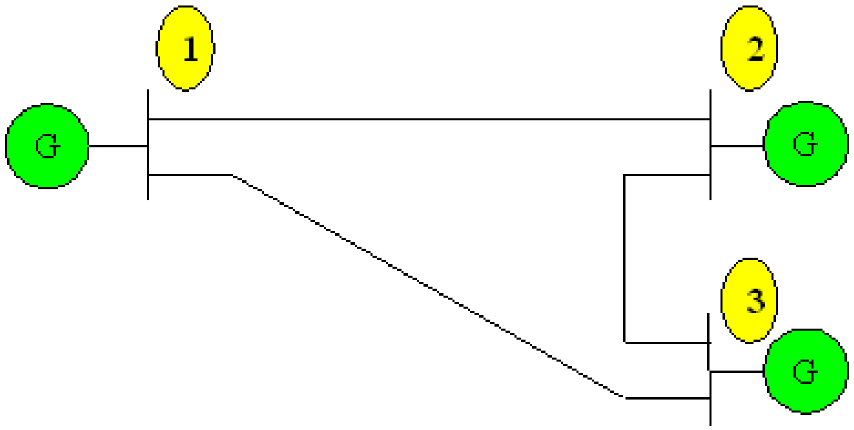


Fig 1.1: Single line diagram of a power system

Sub-graph: Sg is a sub-graph of G if the following conditions are satisfied:

- Sg is itself a graph
- Every node of Sg is also a node of G
- Every branch of Sg is a branch of G



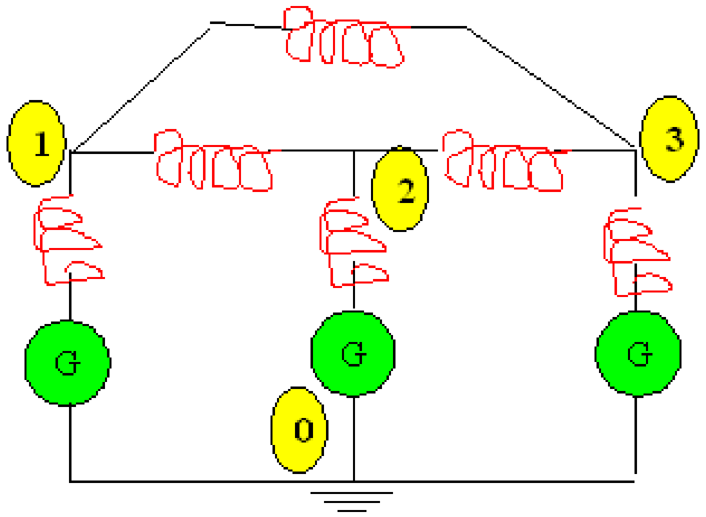


Fig 1.2: Reactance diagram of the given power system in fig 1.1

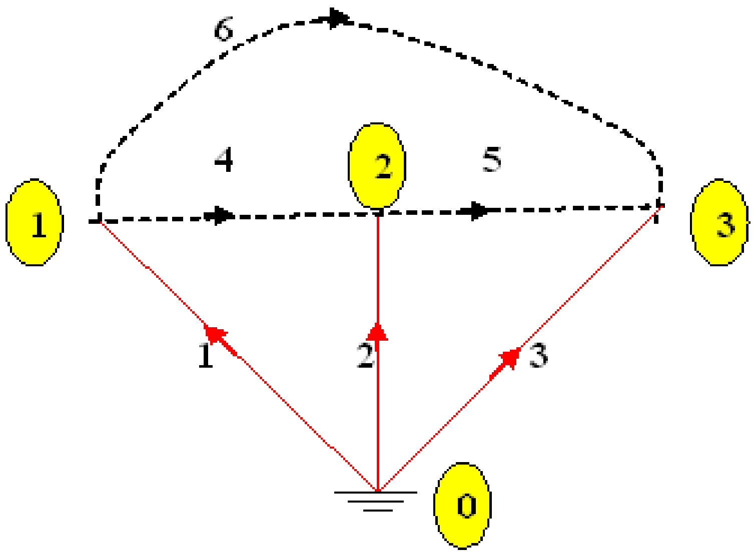


Fig 1.3: Oriented graph

Cutset: It is a set of branches of a connected graph G which satisfies the following conditions:

- The removal of all branches of the cutset causes the remaining graph to have two separate unconnected sub-graphs.
- The removal of all but one of the branches of the set, leaves the remaining graph connected.

Referring to Fig 1.3, the set {3,5,6} constitutes a cutset since removal of them isolates node 3 from rest of the network, thus dividing the graph into two unconnected subgraphs. However, the set(2,4,6) is not a valid cutset! The KCL (Kirchhoff's Current Law) for the cutset is stated as follows: In any lumped network, the algebraic sum of all the branch currents traversing through the given cutset branches is zero.

Tree: It is a connected sub-graph containing all the nodes of the graph G, but without any closed paths (loops). There is one and only one path between every pair of nodes in a tree. The elements of the tree are called twigs or branches. In a graph with *n* nodes,

The number of branches: b = n-1

For the graph of Fig 1.3, some of the possible trees could be T(1,2,3), T(1,4,6), T(2,4,5), T(2,5,6), etc.

Co-Tree : The set of branches of the original graph G, not included in the tree is called the ***co-tree***. The co-tree could be connected or non-connected, closed or open. The branches of the co-tree are called ***links***. By convention, the tree elements are shown as solid lines while the co-tree elements are shown by dotted lines as shown in Fig.1c for tree T(1,2,3). With e as the total number of elements,

The number of links: $l = e - b = e - n + 1$

For the graph of Fig 1.3, the co-tree graphs corresponding to the various tree graphs are as shown in the table below:

Tree	T(1,2,3)	T(1,4,6)	T(2,4,5)	T(2,5,6)
Co-Tree	T(4,5,6)	T(2,3,5)	T(1,3,6)	T(1,3,4)

Basic loops: When a link is added to a tree it forms a closed path or a loop. Addition of each subsequent link forms the corresponding loop. A loop containing only one link and remaining branches is called a *basic loop* or a *fundamental loop*. These loops are defined for a particular tree. Since each link is associated with a basic loop, the number of basic loops is equal to the number of links.

Basic cut-sets: Cut-sets which contain only one branch and remaining links are called *basic cutsets* or fundamental cut-sets. The basic cut-sets are defined for a particular tree. Since each branch is associated with a basic cut-set, the number of basic cut-sets is equal to the number of branches.

MCQ's

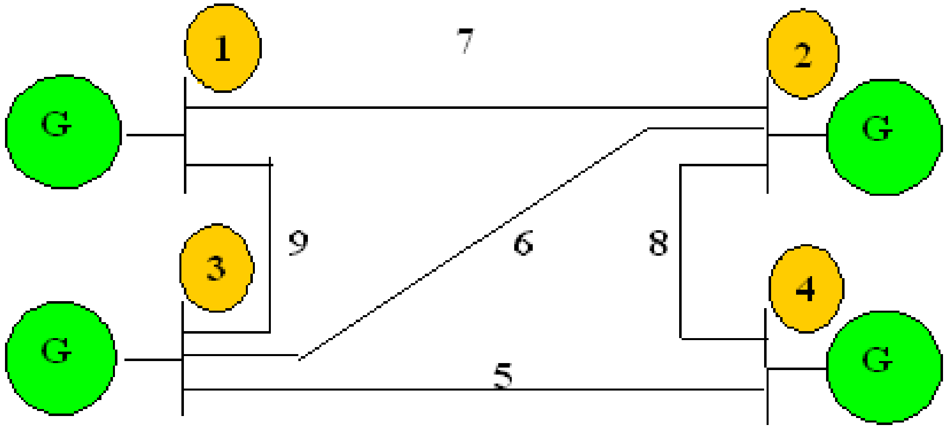
What is the element of the graph that is not included in the tree called?
a. Links
b. Branches
c. Oriented graph
d. All of these

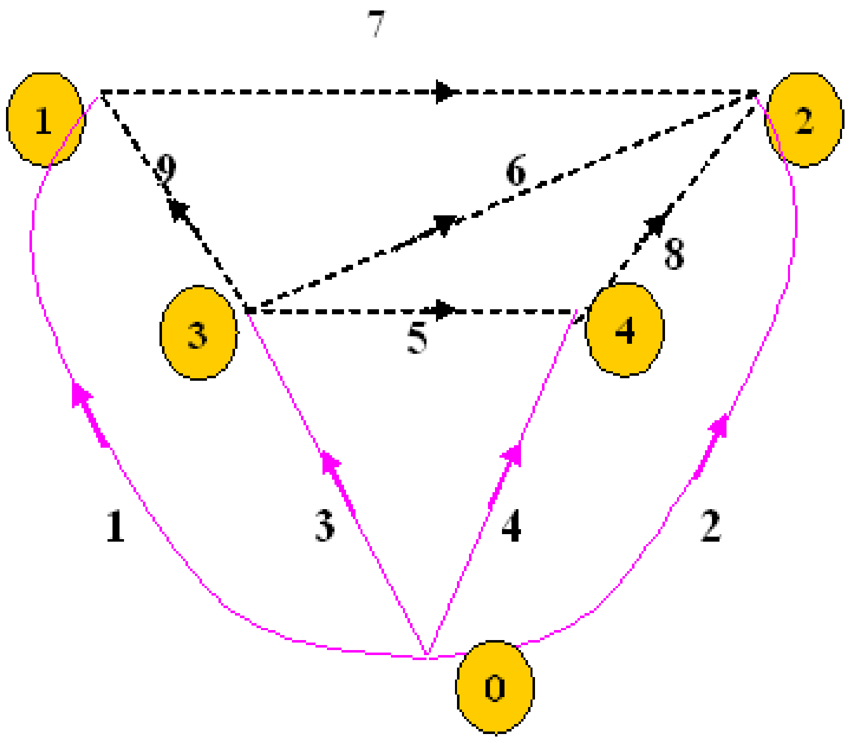
What is an oriented graph?
a. A connection of network topology, represented by replacing all physical elements by lines.
b. A graph in which the direction is assigned to each branch.
c. A graph where at least one path exists between any two nodes of the graph.
d. None of these

Lecture 3

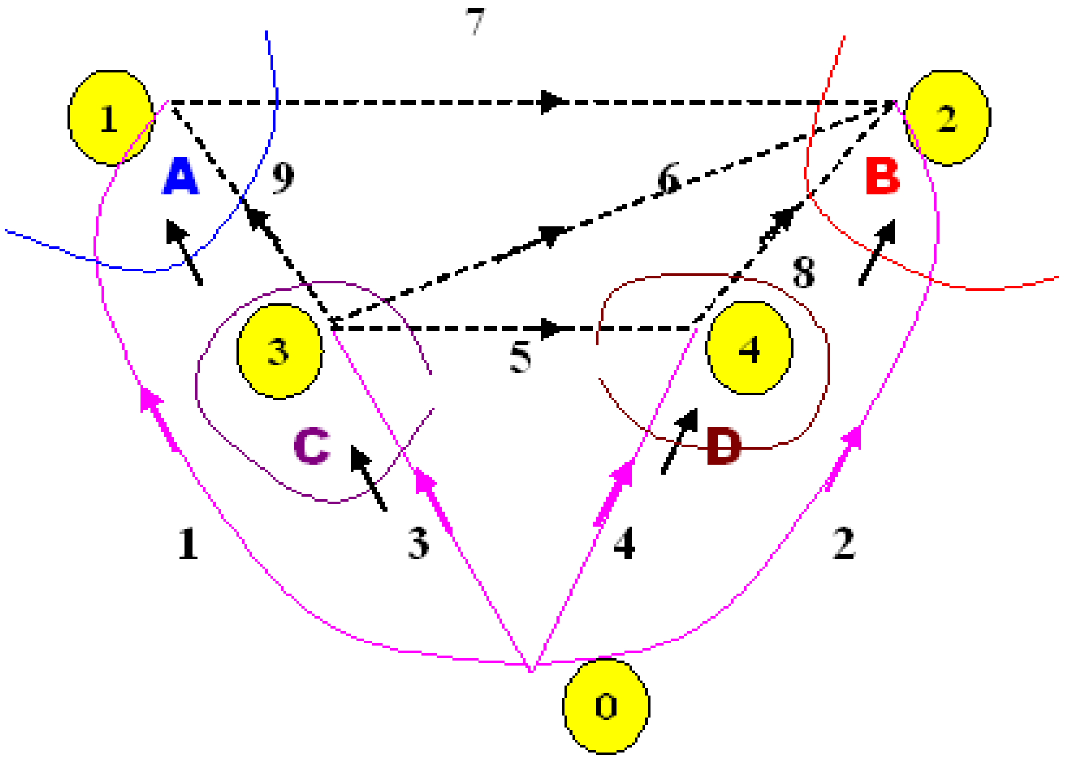
Examples

Example-1: Obtain the oriented graph for the system shown in Fig. below. Select any four possible trees. For a selected tree show the basic loops and basic cut-sets.



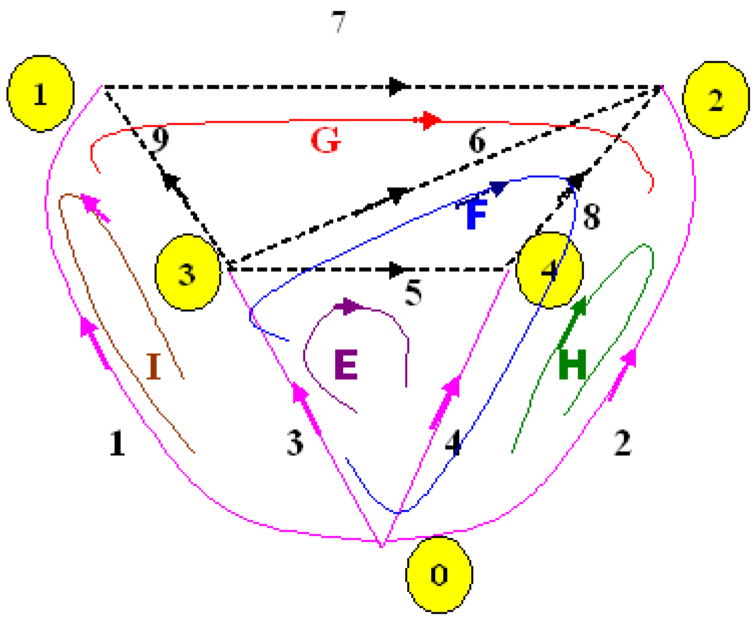


Oriented Graph



Basic Cutsets

For the system given, the oriented graph is as shown in figure E1b. some of the valid Tree graphs could be T(1,2,3,4), T(3,4,8,9), T(1,2,5,6), T(4,5,6,7), etc. The basic cutsets (A,B,C,D) and basic loops (E,F,G,H,I) corresponding to the oriented graph of Fig.E1a and tree, T(1,2,3,4) are as shown in Figures respectively.



Basic Loops

Incidence Matrices

Element–node incidence matrix: $\hat{\mathbf{A}}$

The incidence of branches to nodes in a connected graph is given by the element-node incidence matrix, $\hat{A}$.

An element aij of $\hat{A}$ is defined as under:

aij = 1 if the branch-i is incident to and oriented away from the node-j.

= -1 if the branch-i is incident to and oriented towards the node-j.

= 0 if the branch-i is not at all incident on the node-j.

Thus the dimension of $\hat{A}$ is $e \times n$, where e is the number of elements and n is the number of nodes in the network.

- For example, consider again the sample system with its oriented graph as in fig. 1.3 the corresponding element-node incidence matrix, is obtained as under:

$\hat{A} =$

Nodes / Elements	0	1	2	3
1	1	-1		
2	1		-1	
3	1			-1
4		1	-1	
5			1	-1
6		1		-1

Bus incidence matrix: A

A =

Buses / Elements	1	2	3
1	-1		
2		-1	
3			-1
4	1	-1	
5		1	-1
6	1		-1

=

A_b	Branches
A_l	Links

Examples on Bus Incidence Matrix:

Example-2: For the sample network-oriented graph shown in Fig. E2, by selecting a tree, T(1,2,3,4), obtain the incidence matrices A and Aˆ . Also show the partitioned form of the matrix-A.

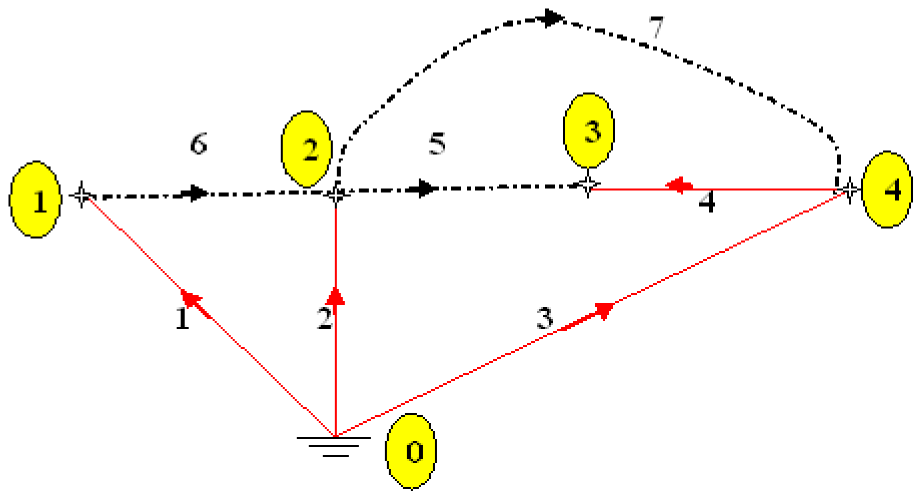


Fig. E2. Sample Network-Oriented Graph

$$\hat{A} = \text{Elements} \begin{array}{c} \text{nodes} \\ \begin{bmatrix} e \backslash n & 0 & 1 & 2 & 3 & 4 \\ 1 & 1 & -1 & 0 & 0 & 0 \\ 2 & 1 & 0 & -1 & 0 & 0 \\ 3 & 1 & 0 & 0 & 0 & -1 \\ 4 & 0 & 0 & 0 & -1 & 1 \\ 5 & 0 & 0 & 1 & -1 & 0 \\ 6 & 0 & 1 & -1 & 0 & 0 \\ 7 & 0 & 0 & 1 & 0 & -1 \end{bmatrix} \end{array}$$

$$\mathbf{A} = \text{Elements} \begin{array}{c} \text{buses} \\ \begin{bmatrix} e \backslash b & 1 & 2 & 3 & 4 \\ 1 & -1 & 0 & 0 & 0 \\ 2 & 0 & -1 & 0 & 0 \\ 3 & 0 & 0 & 0 & -1 \\ 4 & 0 & 0 & -1 & 1 \\ 5 & 0 & 1 & -1 & 0 \\ 6 & 1 & -1 & 0 & 0 \\ 7 & 0 & 1 & 0 & -1 \end{bmatrix} \end{array}$$

Corresponding to the Tree, T(1,2,3,4), matrix-A can be partitioned into two submatrices as under:

$$A_b = \text{branches}\overset{\text{buses}}{\begin{bmatrix} b \backslash b & 1 & 2 & 3 & 4 \\ 1 & -1 & 0 & 0 & 0 \\ 2 & 0 & -1 & 0 & 0 \\ 3 & 0 & 0 & 0 & -1 \\ 4 & 0 & 0 & -1 & 1 \end{bmatrix}}$$

$$A_l = \text{links}\overset{\text{buses}}{\begin{bmatrix} l \backslash b & 1 & 2 & 3 & 4 \\ 5 & 0 & 1 & -1 & 0 \\ 6 & 1 & -1 & 0 & 0 \\ 7 & 0 & 1 & 0 & -1 \end{bmatrix}}$$

MCQ’s

1. Certain graph has 4 nodes and 7 elements, the number of links is:
a) 3
b) 4
c) 11
d) 28

2. The number of nodes and the number of branches in a tree are related by:
a) b = n
b) b = n+1
c) b = n-1
d) b = 2n

Lecture 4

Example-3: For the sample-system shown in Fig. E3, obtain an oriented graph. By selecting a tree, T(1,2,3,4), obtain the incidence matrices A and $\hat{A}$. Also show the partitioned form of the matrix-A.

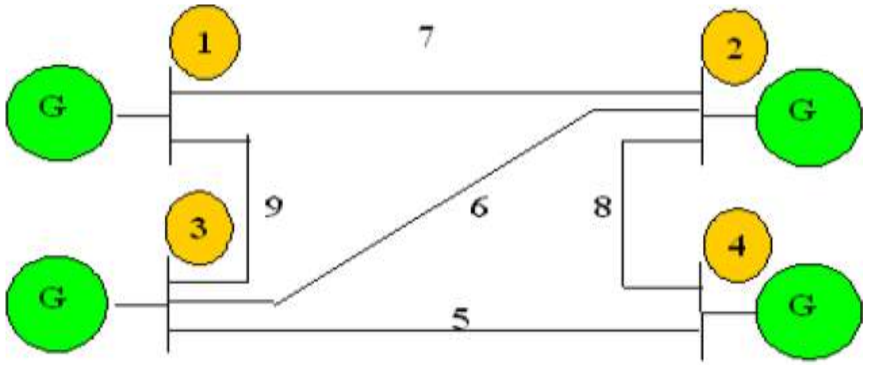


Fig. E3a. Sample Example network

Consider the oriented graph of the given system as shown in figure E3b, below.

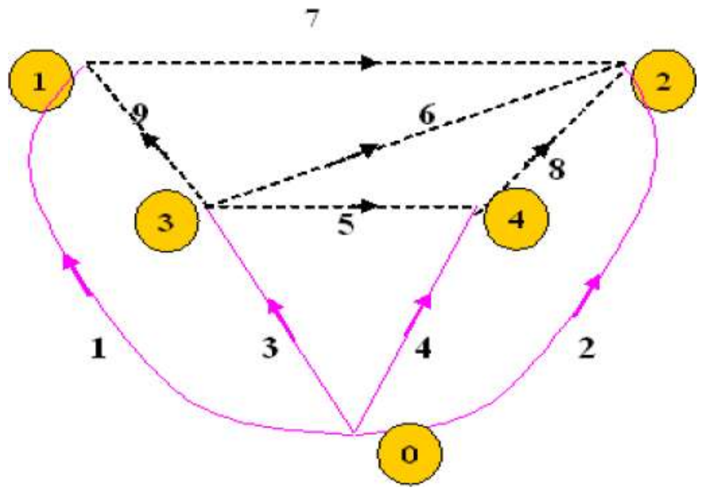


Fig. E3b. Oriented Graph of system of Fig-E3a.

Corresponding to the oriented graph above and a Tree, T(1,2,3,4), the incidence matrices A and A can be obtained as follows:

$\hat{A}$ =

e\n	0	1	2	3	4
1	1	-1			
2	1		-1		
3	1			-1	
4	1				-1
5				1	-1
6			-1	1	
7		1	-1		
8			-1		1
9		-1		1	

A =

e\b	1	2	3	4
1	-1			
2		-1		
3			-1	
4				-1
5			1	-1
6		-1	1	
7	1	-1		
8		-1		1
9	-1		1	

Corresponding to the Tree, T(1,2,3,4), matrix-A can be partitioned into two submatrices as under:

$A_b =$

e\b	1	2	3	4
1	-1			
2		-1		
3			-1	
4				-1

$A_l =$

e\b	1	2	3	4
5			1	-1
6		-1	1	
7	1	-1		
8		-1		1
9	-1		1	

MCQ’s

1. The dimension of bus incidence matrix is:

a. e x n
b. e x (n-1)
c. e x (n+1)
d. e x (n+2)

2. How many fundamental cutsets will be generated for a graph with 'n' number of nodes?

a. n+1
b. n-1
c. $n^2(n-1)$
d. n/ n-1

Lecture 5

Primitive Networks

So far, the matrices of the interconnected network have been defined. These matrices contain complete information about the network connectivity, the orientation of current, the loops and cutsets. However, these matrices contain no information on the nature of the elements which form the interconnected network.

- The complete behaviour of the network can be obtained from the knowledge of the behaviour of the individual elements which make the network, along with the incidence matrices.
- An element in an electrical network is completely characterized by the relationship between the current through the element and the voltage across it.

General representation of a network element: In general, a network element may contain active or passive components. Figure 2 represents the alternative impedance and admittance forms of representation of a general network component

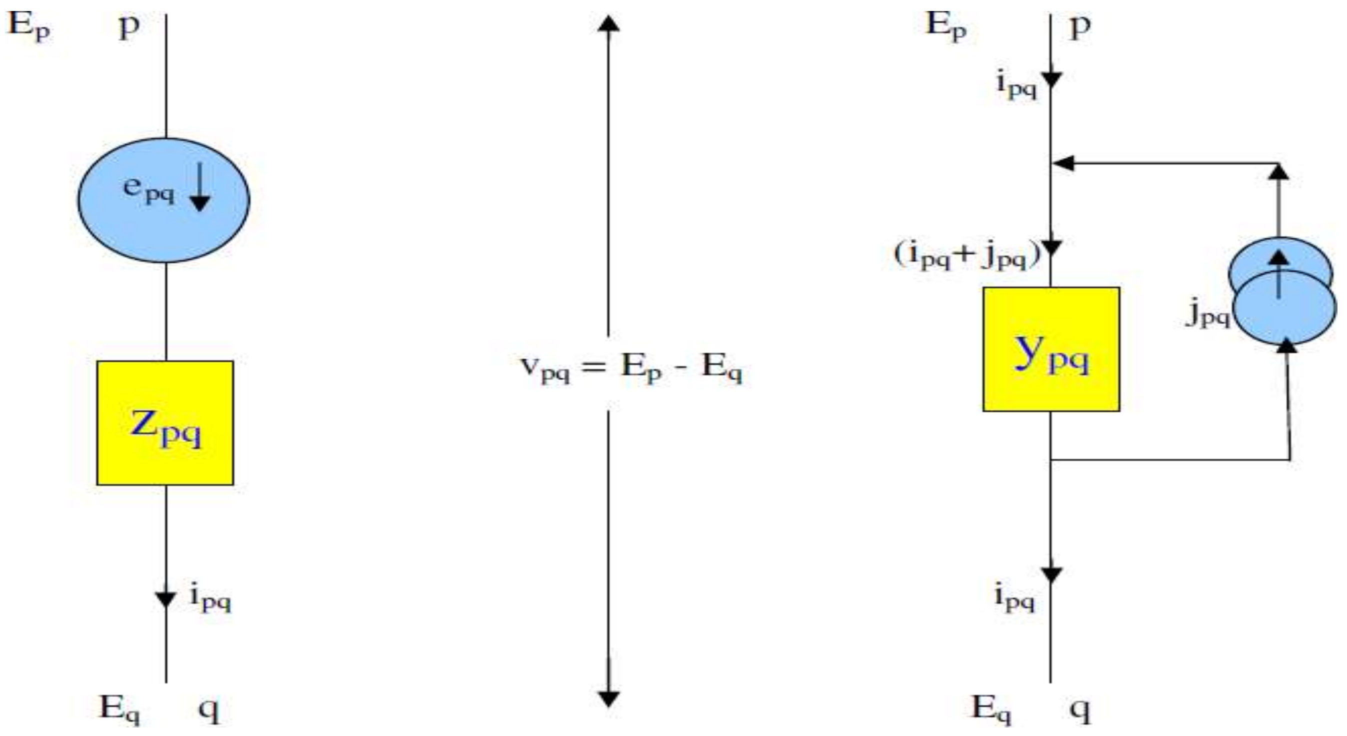


Fig.2 Representation of a primitive network element (a) Impedance form (b) Admittance form

Examples on Primitive Networks:

Example-4: Given that the self impedances of the elements of a network referred by the bus incidence matrix given below are equal to: Z1=Z2=0.2, Z3=0.25, Z4=Z5=0.1 and Z6=0.4 units, draw the corresponding oriented graph, and find the primitive network matrices. Neglect mutual values between the elements.

A =

-1	0	0
0	-1	0
0	0	-1
1	-1	0
0	1	-1
1	0	-1

Example-5: Consider three passive elements whose data is given in Table below. Form the primitive network impedance matrix.

Element number	Self impedance ($z_{pq\text{-}pq}$)		Mutual impedance, ($z_{pq\text{-}rs}$)	
	Bus-code, (p-q)	Impedance in p.u.	Bus-code, (r-s)	Impedance in p.u.
1	1-2	j 0.452		
2	2-3	j 0.387	1-2	j 0.165
3	1-3	j 0.619	1-2	j 0.234

Session 6

Formation of Y_{bus} And Z_{bus}

The bus admittance matrix, Y_{BUS} plays a very important role in computer aided power system analysis. It can be formed in practice by either of the methods as under:

- Rule of Inspection
- Singular Transformation
- Non-Singular Transformation
- ZBUS Building Algorithms, etc.

1. Rule of Inspection

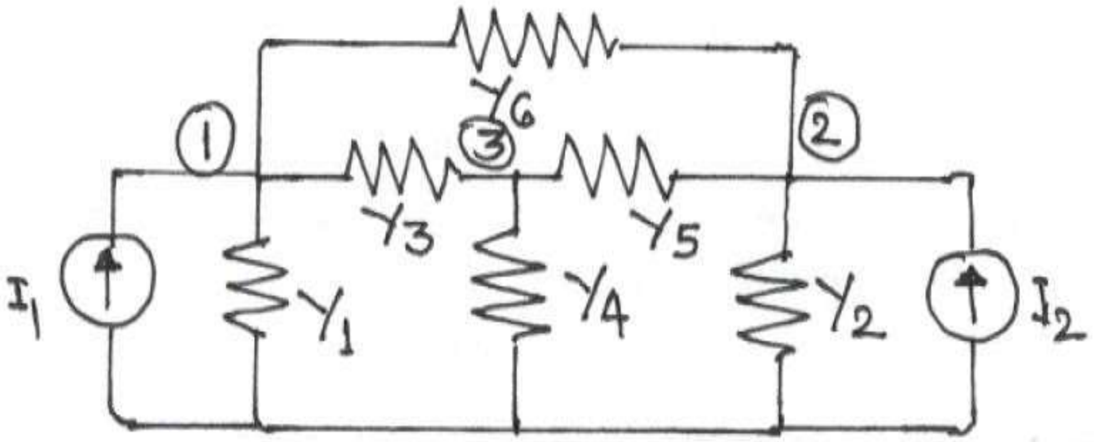
Consider the 3-node admittance network as shown in figure below. Using the basic branch relation:

I = (YV), for all the elemental currents and applying Kirchhoff's Current Law principle at the nodal points, we get the relations as under:

At node 1: I1 =Y1V1 + Y3 (V1-V3) + Y6 (V1 – V2)

At node 2: I2 =Y2V2 + Y5 (V2-V3) + Y6 (V2 – V1`

At node 3: 0 = Y3 (V3-V1) + Y4V3 + Y5 (V3 – V2



These are the performance equations of the given network in admittance form and they can be represented in matrix form as:

$$\begin{vmatrix} I_1 \\ I_2 \\ 0 \end{vmatrix} \begin{matrix} = \\ = \\ = \end{matrix} \begin{vmatrix} (Y_1+Y_3+Y_6) & -Y_6 & -Y_3 \\ -Y_6 & (Y_2+Y_5+Y_6) & -Y_5 \\ -Y_3 & -Y_5 & (Y_3+Y_4+Y_5) \end{vmatrix} \begin{vmatrix} V_1 \\ V_2 \\ V_3 \end{vmatrix}$$

$I_{BUS} = Y_{BUS}\, E_{BUS}$

***Diagonal elements*:** A diagonal element (Yii) of the bus admittance matrix, YBUS, is equal to the sum total of the admittance values of all the elements incident at the bus/node i,

***Off Diagonal elements*:** An off-diagonal element (Yij) of the bus admittance matrix, YBUS, is equal to the negative of the admittance value of the connecting element present between the buses I and j, if any. This is the principle of the rule of inspection. Thus the algorithmic equations for the rule of inspection are obtained as:

Yii = S yij (j = 1,2,…….n)

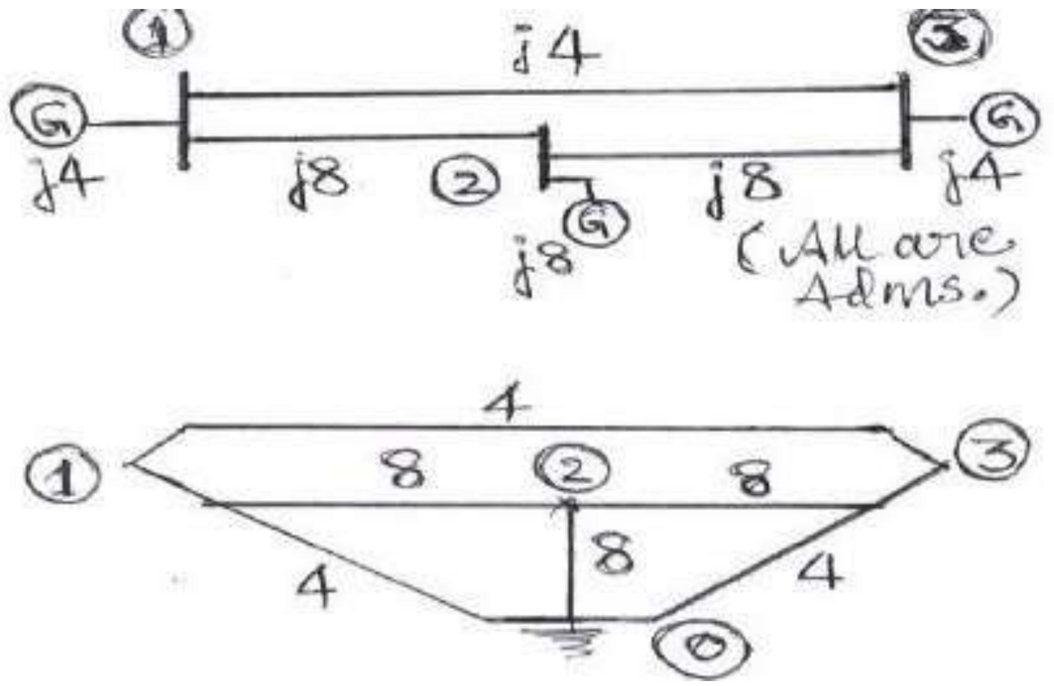
Yij = - yij (j = 1,2,…….n)

For i = 1,2,….n, n = no. of buses of the given system, yij is the admittance of element connected between buses i and j and yii is the admittance of element connected between bus i and ground (reference bus).

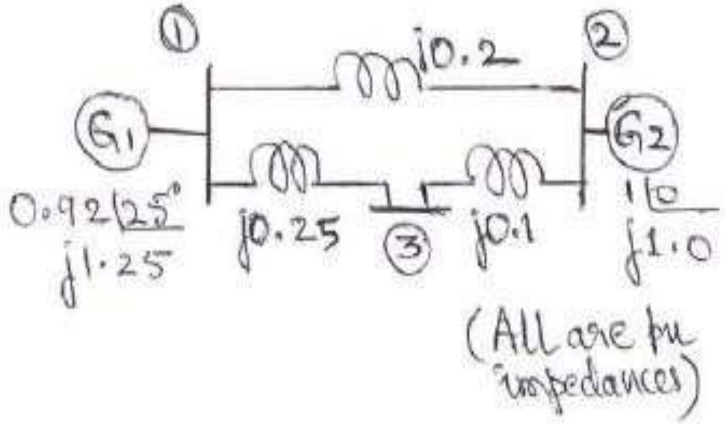
Examples on Rule of Inspection:

Example 6: Obtain the bus admittance matrix for the admittance network shown aside by the rule of inspection

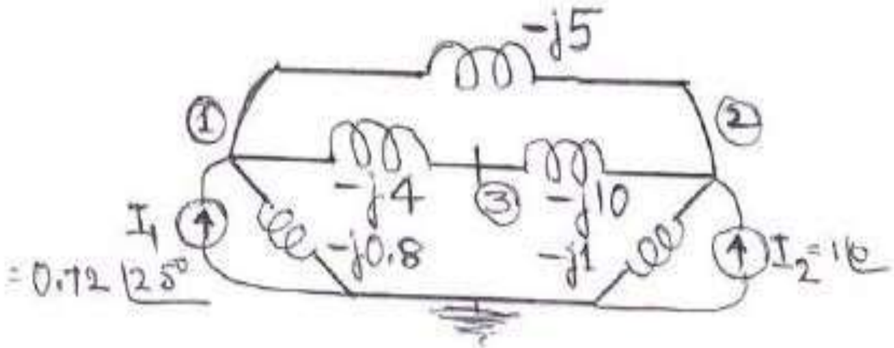
$$Y_{BUS} = j\begin{vmatrix} 16 & -8 & -4 \\ -8 & 24 & -8 \\ -4 & -8 & 16 \end{vmatrix}$$



Example 7: Obtain YBUS for the impedance network shown aside by the rule of inspection. Also, determine YBUS for the reduced network after eliminating the eligible unwanted node. Draw the resulting reduced system diagram.



$$Y_{BUS} = j\begin{vmatrix} -9.8 & 5 & 4 \\ 5 & -16 & 10 \\ 4 & 10 & -14 \end{vmatrix}$$



$$Z_{BUS} = Y_{BUS}^{-1}$$

Session 7

2. Singular Transformations

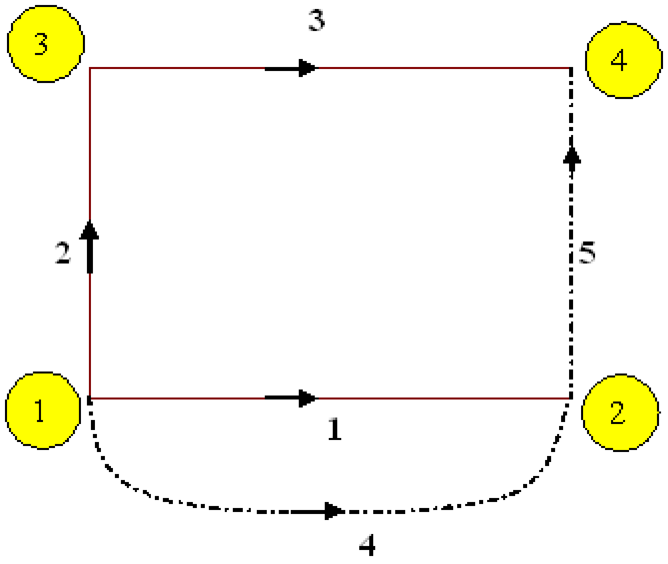
The primitive network matrices are the most basic matrices and depend purely on the impedance or admittance of the individual elements. However, they do not contain any information about the behaviour of the interconnected network variables. Hence, it is necessary to transform the primitive matrices into more meaningful matrices which can relate variables of the interconnected network.

Bus admittance matrix, YBUS and Bus impedance matrix, ZBUS

In the bus frame of reference, the performance of the interconnected network is described by *n* independent nodal equations, where *n* is the total number of buses (n+1nodes are present, out of which one of them is designated as the reference node).

Examples on Singular Transformation:

Example 8: For the network of Fig, form the primitive matrices [z] & [y] and obtain the bus admittance matrix by singular transformation. Choose a Tree T(1,2,3). The data is given in Table.



Elements	Self impedance	Mutual impedance
1	j 0.6	-
2	j 0.5	j 0.1(with element 1)
3	j 0.5	-
4	j 0.4	j 0.2 (with element 1)
5	j 0.2	-

MCQ’s

What is the simplified diagram called, after omitting all resistances, static loads, capacitance of the transmission lines and magnetizing circuit of the transformer?

a. Single line diagram

b. Resistance diagram

c. Reactance diagram

d. Both (a) and (b)

e. None of these

Session 8

Solving Numericals

On Network Topology

Session 9

Solving Numericals

On Network Topology

Session 10

Solving Numericals

On Network Topology

Thank You

Sri Adichunchanagiri Shikshana Trust (R)

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Department of Electrical & Electronics Engineering

Power System Analysis – 2 [18EE71]

Module-2

Load Flow Studies

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SJB Institute of Technology

Lecture 11

Syllabus

Module 2: Load Flow Studies:

Introduction, Classification of buses. Power flow equation, Operating Constraints, Data for Load flow, Gauss Seidal iterative method. Illustrative examples.

Introduction

- Load flow studies are important in planning and designing future expansion of power systems. The study gives steady state solutions of the voltages at all the buses, for a particular load condition.
- Generally, load flow studies are limited to the transmission system, which involves bulk power transmission. The load at the buses is assumed to be known.
- Load flow studies throw light on some of the important aspects of the system operation, such as: violation of voltage magnitudes at the buses, overloading of lines, overloading of generators, stability margin reduction, indicated by power angle differences between buses linked by a line, effect of contingencies like line voltages, emergency shutdown of generators, etc.

Introduction Continued..

- Load flow studies are required for deciding the economic operation of the power system.
- The load flow problem consists of finding the power flows (real and reactive) and voltages of a network for given bus conditions.
- At each bus, there are four quantities of interest to be known for further analysis: the real and reactive power, the voltage magnitude and its phase angle.
- Because of the nonlinearity of the algebraic equations, describing the given power system, their solutions are obviously, based on the iterative methods only.

MCQ’s

1. The power system analysis is essential for

a)Planning the operation

b)Improvement and expansion of power system

c)Both a and b

d)None of the above

Lecture 12

Classification of buses

Sl. No.	Bus Types	Specified Variables	Unspecified variables	Remarks
1	Slack/ Swing Bus	$\|V\|$, δ	P_G, Q_G	$\|V\|$, δ: are assumed if not specified as 1.0 and 0^0
2	Generator/ Machine/ PV Bus	P_G, $\|V\|$	Q_G, δ	A generator is present at the machine bus
3	Load/ PQ Bus	P_G, Q_G	$\|V\|$, δ	About 80% buses are of PQ type
4	Voltage Controlled Bus	P_G, Q_G, $\|V\|$	δ, a	'a' is the % tap change in tap-changing transformer

The Load Flow Problem and Power Flow Equations

- Here, the analysis is restricted to a balanced three-phase power system, so that the analysis can be carried out on a single phase basis. The per unit quantities are used for all quantities.

- The first step in the analysis is the formulation of suitable equations for the power flows in the system.

- At any bus, complex power is injected into the bus by the generators and complex power is drawn by the loads. Of course at any bus, either one of them may not be present.

- The power is transported from one bus to other via the transmission lines. At any bus *i*, the complex power Si (injected), shown in figure 1, is defined as

$$S_i = S_{Gi} - S_{Di} \tag{2}$$

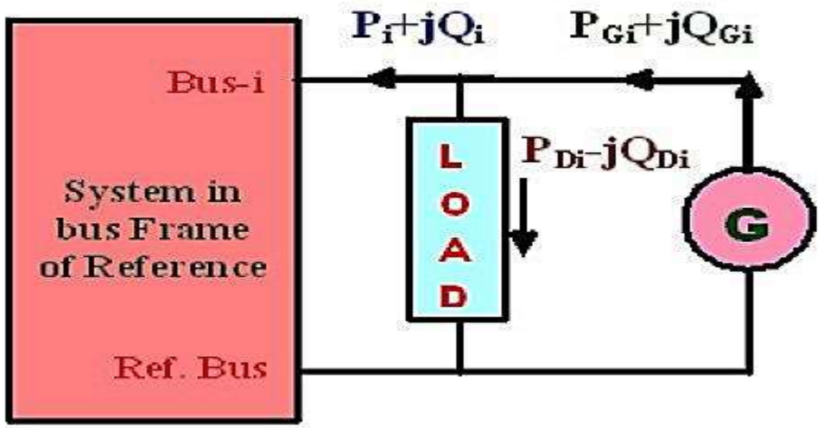

Fig.1 power flows at a bus-i

$I_{BUS} = Y_{BUS} V_{BUS}$

where

$$I_{BUS} = \begin{bmatrix} I_1 \\ I_2 \\ . \\ . \\ I_n \end{bmatrix}$$ is the vector of currents injected at the buses,

Y_{BUS} is the bus admittance matrix, and

$$V_{BUS} = \begin{bmatrix} V_1 \\ V_2 \\ . \\ . \\ V_n \end{bmatrix}$$ is the vector of complex bus voltages.

Equation (5) can be considered as

$$I_i = \sum_{j=1}^{n} Y_{ij} V_j \qquad \forall\, i = 1, 2, \ldots\ldots\ldots n \tag{6}$$

The complex power S_i is given by

$$S_i = V_i\, I_i^*$$

$$= V_i \left(\sum_{j=1}^{n} Y_{ij}\, V_j \right)^*$$

$$= V_i \left(\sum_{j=1}^{n} Y_{ij}^* V_j^* \right) \tag{7}$$

Let $\qquad V_i \triangleq |V_i| \angle\, \delta_i = |V_i| (\cos\, \delta_i + j \sin\, \delta_i)$

$$\delta_{ij} = \delta_i - \delta_j$$

$$Y_{ij} = G_{ij} + jB_{ij}$$

Hence from (7), we get,

$$S_i = \sum_{j=1}^{n} |V_i| \, |V_j| \left(\cos \delta_{ij} + j \sin \delta_{ij}\right) \left(G_{ij} - j B_{ij}\right) \qquad \textbf{(8)}$$

Separating real and imaginary parts in (8) we obtain,

$$P_i = \sum_{j=1}^{n} |V_i| \, |V_j| \left(G_{ij} \cos \delta_{ij} + B_{ij} \sin \delta_{ij}\right) \qquad \textbf{(9)}$$

$$Q_i = \sum_{j=1}^{n} |V_i| \, |V_j| \left(G_{ij} \sin \delta_{ij} - B_{ij} \cos \delta_{ij}\right) \qquad \textbf{(10)}$$

An alternate form of P_i and Q_i can be obtained by representing Y_{ik} also in polar form

as $\qquad Y_{ij} = |Y_{ij}| \angle \theta_{ij} \qquad$ **(11)**

Again, we get from (7),

$$S_i = |V_i| \angle \delta_i \sum_{j=1}^{n} |Y_{ij}| \angle -\theta_{ij} \, |V_j| \angle -\delta_j \qquad \textbf{(12)}$$

Again, we get from (7),

$$S_i = |V_i|\angle\delta_i \sum_{j=1}^{n} |Y_{ij}|\angle-\theta_{ij}\, |V_j|\angle-\delta_j \qquad \textbf{(12)}$$

The real part of (12) gives P_i.

$$P_i = |V_i| \sum_{j=1}^{n} |Y_{ij}|\, |V_j| \cos\left(-\theta_{ij} + \delta_i - \delta_j\right)$$

$$= |V_i| \sum_{j=1}^{n} |Y_{ij}|\, |V_j| \cos-(\theta_{ij} - \delta_i + \delta_j) \qquad \text{or}$$

$$P_i = \sum_{j=1}^{n} |V_i||V_j||Y_{ij}| \cos(\theta_{ij} - \delta_i + \delta_j) \qquad \forall\ \mathrm{i} = 1, 2,\ldots\ldots\ldots\mathrm{n}, \qquad \textbf{(13)}$$

Similarly, Q_i is imaginary part of (12) and is given by

$$Q_i = |V_i| \sum_{j=1}^{n} |Y_{ij}|\, |V_j| \sin-(\theta_{ij} - \delta_i + \delta_j) \qquad \text{or}$$

$$Q_i = -\sum_{j=1}^{n} |V_i||V_j||Y_{ij}| \sin(\theta_{ij} - \delta_i + \delta_j) \qquad \forall\ \mathrm{i} = 1, 2,\ldots\ldots\ldots\mathrm{n} \qquad \textbf{(14)}$$

Lecture 13

Data for Load Flow

The various data required are as under:

1. **System data**:
   - It includes:
   - Number of buses-*n*,
   - Number of PV buses,
   - Number of loads,
   - Number of transmission lines,
   - Number of transformers,
   - Number of shunt elements,
   - The slack bus number,
   - Voltage magnitude of slack bus (angle is generally taken as 0o),
   - Tolerance limit,
   - Base MVA and
   - Maximum permissible number of iterations.

2. Generator bus data: For every PV bus *i*, the data required includes the

- Bus number,
- Active power generation P_{Gi},
- The specified voltage magnitude
- Minimum reactive power limit Qi,min, and
- Maximum reactive power limit Qi,max.

3. Load data: For all loads the data required includes the

- Bus number,
- Active power demand PDi, and
- The reactive power demand QDi.

4. Transmission line data: For every transmission line connected between buses *i* and *k* the data includes the

- Starting bus number *i*,
- Ending bus number *k*,
- *R*esistance of the line,
- Reactance of the line and
- Half line charging admittance.

5. Transformer data: For every transformer connected between buses *i* and *k* the data to be given includes:

- Starting bus number *i*,
- Ending bus number *k*,
- Resistance of the transformer,
- Reactance of the transformer, and
- The off nominal turns-ratio *a*.

6. Shunt element data: The data needed for the shunt element includes:

- The bus number where element is connected, and
- The shunt admittance (Gsh + j Bsh).

MCQ’s

What is the element of the graph that is not included in the tree called?
a. Links
b. Branches
c. Oriented graph
d. All of these

What is an oriented graph?
a. A connection of network topology, represented by replacing all physical elements by lines.
b. A graph in which the direction is assigned to each branch.
c. A graph where at least one path exists between any two nodes of the graph.
d. None of these

Lecture 14

Gauss – Seidel (GS) Method

The GS method is an iterative algorithm for solving nonlinear algebraic equations. An initial solution vector is assumed, chosen from past experiences, statistical data or from practical considerations. At every subsequent iteration, the solution is updated till convergence is reached.

Case (a): Systems with PQ buses only:

Initially assume all buses to be PQ type buses, except the slack bus. This means that (n–1) complex bus voltages have to be determined. For ease of programming, the slack bus is generally numbered as bus-1. PV buses are numbered in sequence and PQ buses are ordered next in sequence. This makes programming easier, compared to random ordering of buses. Consider the expression for the complex power at bus-i, given from Equation (7), as:

$$S_i = V_i \left(\sum_{j=1}^{n} Y_{ij} V_j \right)^*$$

This can be written as

$$S_i^* = V_i^* \left(\sum_{j=1}^{n} Y_{ij} V_j \right) \qquad (15)$$

Since $S_i^* = P_i - jQ_i$, we get,

$$\frac{P_i - jQ_i}{V_i^*} = \sum_{j=1}^{n} Y_{ij} V_j$$

So that,

$$\frac{P_i - jQ_i}{V_i^*} = Y_{ii} V_i + \sum_{\substack{j=1 \\ j \neq i}}^{n} Y_{ij} V_j \qquad (16)$$

Rearranging the terms, we get,

$$V_i = \frac{1}{Y_{ii}} \left[\frac{P_i - jQ_i}{V_i^*} - \sum_{\substack{j=1 \\ j \neq i}}^{n} Y_{ij} V_j \right] \quad \forall\, i = 2, 3, \ldots\ldots\ldots n \qquad (17)$$

Lecture 15

Algorithm for GS method

1. Prepare data for the given system as required.
2. Formulate the bus admittance matrix YBUS. This is generally done by the rule of inspection.
3. Assume initial voltages for all buses, 2,3,…n. In practical power systems, the magnitude of the bus voltages is close to 1.0 p.u. Hence, the complex bus voltages at all (n-1) buses (except slack bus) are taken to be 1.0□0^0. This is normally refered as the ***flat start*** solution.
4. Update the voltages. In any (k +1)st iteration, from (17) the voltages are given by

$$V_i^{(k+1)} = \frac{1}{Y_{ii}}\left[\frac{P_i - jQ_i}{(V_i^{(k)})^*} - \sum_{j=1}^{i-1} Y_{ij} V_j^{(k+1)} - \sum_{j=i+1}^{n} Y_{ij} V_j^{(k)}\right] \forall\ i=2,3,\ldots n \qquad (18)$$

$$\left|\Delta V_i^{(k+1)}\right| = \left|V_i^{(k+1)} - V_i^{(k)}\right| < \varepsilon \qquad \forall\ i = 2,3,\ldots n \qquad (19)$$

Where, ε is the tolerance value. Generally it is customary to use a value of 0.0001 pu. Compute slack bus power after voltages have converged using (15) [assuming bus 1 is slack bus].

5. Compute all line flows.

6. The complex power loss in the line is given by Sik + Ski. The total loss in the system is calculated by summing the loss over all the lines.

$$S_1^* = P_1 - jQ_1 = V_1^*\left(\sum_{j=1}^{n} Y_{1j} V_j\right) \qquad (20)$$

Case (b): Systems with PV buses also present:

At PV buses, the magnitude of voltage and not the reactive power is specified. Hence it is needed to first make an estimate of Qi to be used in (18). From (15) we have

$$Q_i = -\,\mathrm{Im}\left\{V_i^* \sum_{j=1}^{n} Y_{ij}\, V_j\right\}$$

Where *Im* stands for the imaginary part. At any *(k+1)st* iteration, at the PV bus-*i*,

$$Q_i^{(k+1)} = -\,\mathrm{Im}\left\{(V_i^{(k)})^* \sum_{j=1}^{i-1} Y_{ij}\, V_j^{(k+1)} + (V_i^{(k)})^* \sum_{j=i}^{n} Y_{ij}\, V_j^{(k)}\right\} \qquad \textbf{(21)}$$

The steps for i^{th} PV bus are as follows:

1. Compute $Q_i^{(k+1)}$ using (21)
2. Calculate V_i using (18) with $Q_i = Q_i^{(k+1)}$
3. Since $|V_i|$ is specified at the PV bus, the magnitude of V_i obtained in step 2

has to be modified and set to the specified value $|V_{i,sp}|$. Therefore,

$$V_i^{(k+1)} = |V_{i,sp}| \angle \delta_i^{(k+1)} \tag{22}$$

The voltage computation for PQ buses does not change.

Case (c): Systems with PV buses with reactive power generation limits specified:

In the previous algorithm if the Q limit at the voltage controlled bus is violated during any iteration, i.e $(k+1)$ i Q computed using (21) is either less than Qi, min or greater than Qi,max, it means that the voltage cannot be maintained at the specified value due to lack of reactive power support. This bus is then treated as a PQ bus in the *(k+1)st* iteration and the voltage is calculated with the value of Qi set as follows

If $Q_i < Q_{i,min}$ Then $Q_i = Q_{i,min}$.

If $Q_i > Q_{i,max}$ Then $Q_i = Q_{i,max}$.

(23)

Acceleration of convergence

It is found that in GS method of load flow, the number of iterations increase with increase in the size of the system. The number of iterations required can be reduced if the correction in voltage at each bus is accelerated, by multiplying with a constant α, called the acceleration factor. In the (k+1)st iteration we can let

$$V_i^{(k+1)}(accelerate\ d) = V_i^{(k)} + \alpha\left(V_i^{(k+1)} - V_i^{(k)}\right) \qquad (24)$$

where α is a real number. When α =1, the value of (*k* +1) is the computed value. If 1<α<2 then the value computed is extrapolated. Generally α is taken between 1.6 to 2.0, for GS load flow procedure. At PQ buses (pure load buses) if the voltage magnitude violates the limit, it simply means that the specified reactive power demand cannot be supplied, with the voltage maintained within acceptable limits.

MCQ's

1. Certain graph has 4 nodes and 7 elements, the number of links is:

a) 3
b) 4
c) 11
d) 28

2. The number of nodes and the number of branches in a tree are related by:

a) $b = n$
b) $b = n+1$
c) $b = n-1$
d) $b = 2n$

Lecture 16

Solving Numericals

On GS Method

Lecture 17

Solving Numericals

On GS Method

Lecture 18

Solving Numericals

On GS Method

Lecture 19

Solving Numericals

On GS Method

Lecture 20

Solving Numericals

On GS Method

Thank You

Sri Adichunchanagiri Shikshana Trust (R)

SJB Institute of Technology

(Affiliated to Visvesvaraya Technological University, Belagavi& Approved by AICTE, New Delhi.)

Department of Electrical & Electronics Engineering

Power System Analysis – 2 [18EE71]

Module-3

Load Flow Studies

By:

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Lecture 21

Syllabus

Module 3: Load Flow Studies (Continued):

Newton-Raphson method derivation in Polar form, Fast decoupled load flow method, Flow charts of LFS methods. Comparison of Load Flow Methods. Illustrative examples.

Newton-Raphson Method

- Although the Gauss-Seidel was the first popular method for load flow calculations, the Newton-Raphson method is now commonly used.

- The Newton-Raphson (NR) method has better convergence characteristics and for many systems is faster than the Gauss-Seidel method; the former has a much larger time per iteration but requires very few iterations (four is general), whereas the Gauss-Siedel requires up to 30 iterations, the number increasing with the size of system

NR method is used to solve a system of non-linear algebraic equations of the form f(x)=0.

Consider a set of *n* non-linear algebraic equations given by

$$f_i(x_1, x_2 \ldots\ldots x_n) = 0 \qquad i = 1,2\ldots\ldots n \qquad (25)$$

Let $x_1^0, x_2^0 \ldots\ldots\ldots x_n^0$, be the initial guess of unknown variables and $\Delta x_1^0, \Delta x_2^0 \ldots\ldots \Delta x_n^0$ be the respective corrections. Therefore,

$$f_i(x_1^0 + \Delta x_1^0, x_2^0 + \Delta x_2^0 \ldots\ldots\ldots x_n^0 + \Delta x_n^0) = 0 \qquad i = 1,2\ldots\ldots n \qquad (26)$$

The above equation can be expanded using Taylor's series to give

$$f_i(x_1^0, x_2^0 \ldots\ldots x_n^0) + \left[\left(\frac{\partial f_i}{\partial x_1}\right)^0 \Delta x_1^0 + \left(\frac{\partial f_i}{\partial x_2}\right)\Delta x_2^0 + \ldots\ldots + \left(\frac{\partial f_i}{\partial x_n}\right)\Delta x_n^0\right]$$

$$+ \text{Higher order terms} = 0 \qquad \forall\ i = 1,2\ldots\ldots n \qquad (27)$$

Where, $\left(\frac{\partial f_i}{\partial x_1}\right)^0, \left(\frac{\partial f_i}{\partial x_2}\right)^0, \ldots\ldots\ldots\ldots \left(\frac{\partial f_i}{\partial x_n}\right)^0$ are the partial derivatives of f_i with respect to $x_1, x_2 \ldots\ldots x_n$ respectively, evaluated at $(x_1^0, x_2^0 \ldots\ldots\ldots x_0^n)$. If the higher order terms are neglected, then (27) can be written in matrix form as to $x_1, x_2 \ldots\ldots x_n$ respectively, evaluated at $(x_1^{\,0}, x_2^{\,0} \ldots\ldots\ldots x_0^{\,n})$. If the higher order terms are neglected, then (27) can be written in matrix form as

$$\begin{bmatrix} f_1^{\,0} \\ f_2^{\,0} \\ \cdot \\ \cdot \\ \cdot \\ f_n^{\,0} \end{bmatrix} + \begin{bmatrix} \left(\frac{\partial f_1}{\partial x_1}\right)^0 & \left(\frac{\partial f_1}{\partial x_2}\right)^0 & \cdot\;\cdot\;\cdot & \left(\frac{\partial f_1}{\partial x_n}\right)^0 \\ \left(\frac{\partial f_2}{\partial x_1}\right)^0 & \left(\frac{\partial f_2}{\partial x_2}\right)^0 & \cdot\;\cdot\;\cdot & \left(\frac{\partial f_2}{\partial x_n}\right)^0 \\ \cdot & \cdot & \cdot\;\cdot\;\cdot & \cdot \\ \cdot & \cdot & \cdot\;\cdot\;\cdot & \cdot \\ \cdot & \cdot & \cdot\;\cdot\;\cdot & \cdot \\ \left(\frac{\partial f_n}{\partial x_1}\right) & \left(\frac{\partial f_n}{\partial x_2}\right) & \cdot\;\cdot\;\cdot & \left(\frac{\partial f_n}{\partial x_n}\right) \end{bmatrix} \begin{bmatrix} \Delta x_1^{\,0} \\ \Delta x_2^{\,0} \\ \cdot \\ \cdot \\ \cdot \\ \Delta x_n^{\,0} \end{bmatrix} = 0 \qquad \textbf{(28)}$$

In vector form (28) can be written as

$$F^0 + J^0 \Delta X^0 = 0$$

Or $$F^0 = -J^0 \Delta X^0$$

Or $$\Delta X^0 = -[J^0]^{-1} F^0 \qquad \textbf{(29)}$$

And $$X^1 = X^0 + \Delta X^0 \qquad \textbf{(30)}$$

In vector form (28) can be written as

$$F^0 + J^0 \Delta X^0 = 0$$

Or $$F^0 = -J^0 \Delta X^0$$

Or $$\Delta X^0 = -[J^0]^{-1} F^0 \tag{29}$$

And $$X^1 = X^0 + \Delta X^0 \tag{30}$$

Here, the matrix [J] is called the Jacobian matrix

MCQ’s

1. The power system analysis is essential for

a)Planning the operation

b)Improvement and expansion of power system

c)Both a and b

d)None of the above

Lecture 22

NR method for load flow solution in polar coordinates

In application of the NR method, we have to first bring the equations to be solved, to the form $f_i(x_1, x_2, ... x_n) = 0$, where $x_1, x_2, ... x_n$ are the unknown variables to be determined. Let us assume that the power system has n_1 PV buses and n_2 PQ buses. In polar coordinates the unknown variables to be determined are:

(i) δ_i, the angle of the complex bus voltage at bus i, at all the PV and PQ buses. This gives us $n_1 + n_2$ unknown variables to be determined.

(ii) $|V_i|$, the voltage magnitude of bus i, at all the PQ buses. This gives us n_2 unknown variables to be determined.

Therefore, the total number of unknown variables to be computed is: $n_1 + 2n_2$, for which we need $n_1 + 2n_2$ consistent equations to be solved. The equations are given by,

$$\Delta P_i = P_{i,sp} - P_{i,cal} = 0 \qquad \textbf{(31)}$$

$$\Delta Q_i = Q_{i,sp} - Q_{i,cal} = 0 \qquad \textbf{(32)}$$

Where $P_{i,sp}$ = Specified active power at bus ***i***

$Q_{i,sp}$ = Specified reactive power at bus ***i***

$P_{i,cal}$ = Calculated value of active power using voltage estimates.

$Q_{i,cal}$ = Calculated value of reactive power using voltage estimates

ΔP = Active power residue

ΔQ = Reactive power residue

The real power is specified at all the PV and PQ buses. Hence (31) is to be solved at all PV and PQ buses leading to $n_1 + n_2$ equations. Similarly the reactive power is specified at all the PQ buses. Hence, (32) is to be solved at all PQ buses leading to n_2 equations.

We thus have $n_1 + 2n_2$ equations to be solved for $n_1 + 2n_2$ unknowns. (31) and (32 are of the form F(x) = 0. Thus NR method can be applied to solve them. Equation: (31) and (32) can be written in the form of (30) as:

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} J_1 & J_2 \\ J_3 & J_4 \end{bmatrix} \begin{bmatrix} \Delta \delta \\ \Delta |V| \end{bmatrix} \quad (33)$$

Where J_1, J_2, J_3, J_4 are the negated partial derivatives of ΔP and ΔQ with respec to corresponding δ and $|V|$. The negated partial derivative of ΔP, is same as the partia derivative of P_{cal}, since P_{sp} is a constant. The various computations involved are discussed in detail next.

Computation of P_{cal} and Q_{cal}:

The real and reactive powers can be computed from the load flow equations as:

$$P_{i,Cal} = P_i = \sum_{k=1}^{n} |V_i||V_k|(G_{ik}\cos\delta_{ik} + B_{ik}\sin\delta_{ik})$$

$$= G_{ii}|V_i|^2 + \sum_{\substack{k=1 \\ k\neq i}}^{n} |V_i||V_k|(G_{ik}\cos\delta_{ik} + B_{ik}\sin\delta_{ik}) \qquad (34)$$

$$Q_{i,Cal} = Q_i = \sum_{k=1}^{n} |V_i||V_k|(G_{ik}\sin\delta_{ik} - B_{ik}\cos\delta_{ik})$$

$$= -B_{ii}|V_i|^2 + \sum_{\substack{k=1 \\ k\neq i}}^{n} |V_i||V_k|(G_{ik}\sin\delta_{ik} - B_{ik}\cos\delta_{ik}) \qquad (35)$$

Elements of J_1

$$\frac{\partial P_i}{\partial \delta_i} = \sum_{\substack{k=1 \\ k \neq i}}^{n} |V_i||V_k| \{ G_{ik}(-\sin\delta_{ik}) + B_{ik}\cos\delta_{ik} \}$$

$$= -Q_i - B_{ii}|V_i|^2$$

$$\frac{\partial P_i}{\partial \delta_k} = |V_i||V_k| (G_{ik}(-\sin\delta_{ik})(-1) + B_{ik}(\cos\delta_{ik})(-1))$$

Elements of J_3

$$\frac{\partial Q_i}{\partial \delta_i} = \sum_{\substack{k=1 \\ k \neq i}}^{n} |V_i||V_k|(G_{ik}\cos\delta_{ik} + B_{ik}\sin\delta_{ik}) = P_i - G_{ii}|V_i|^2$$

$$\frac{\partial Q_i}{\partial \delta_k} = -|V_i||V_k|(G_{ik}\cos\delta_{ik} + B_{ik}\sin\delta_{ik})$$

Elements of J_2

$$\frac{\partial P_i}{\partial |V_i|}|V_i| = 2|V_i|^2 G_{ii} + |V_i|\sum_{\substack{k=1 \\ k \neq i}}^{n} |V_k|(G_{ik}\cos\delta_{ik} + B_{ik}\sin\delta_{ik}) = P_i + |V_i|^2 G$$

$$\frac{\partial P_i}{\partial |V_k|}|V_k| = |V_i||V_k|(G_{ik}\cos\delta_{ik} + B_{ik}\sin\delta_{ik})$$

Elements of J_4

$$\frac{\partial P_i}{\partial |V_i|}|V_i| = -2|V_i|^2 B_{ii} + \sum_{\substack{k=1 \\ k\neq i}}^{n} |V_i||V_k|(G_{ik}\sin\delta_{ik} - B_{ik}\cos\delta_{ik}) = Q_i - |V_i|^2$$

$$\frac{\partial Q_i}{\partial |V_k|}|V_k| = |V_i||V_k|(G_{ik}\sin\delta_{ik} - B_{ik}\cos\delta_{ik})$$

Thus, the linearized form of the equation could be considered agai

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} H & N \\ M & L \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \dfrac{\Delta|V|}{|V|} \end{bmatrix}$$

The elements are summarized below:

(i) $H_{ii} = \dfrac{\partial P_i}{\partial \delta_i} = -Q_i - B_{ii}|V_i|^2$

(ii) $H_{ik} = \dfrac{\partial P_i}{\partial \delta_k} = a_k f_i - b_k e_i = |V_i||V_k|(G_{ik} \sin \delta_{ik} - B_{ik} \cos \delta_{ik})$

(iii) $N_{ii} = \dfrac{\partial P_i}{\partial |V_i|}|V_i| = P_i + G_{ii}|V_i|^2$

(iv) $N_{ik} = \dfrac{\partial P_i}{\partial |V_k|}|V_k| = a_k e_i + b_k f_i = |V_i||V_k|(G_{ik} \cos \delta_{ik} + B_{ik} \sin \delta_{ik})$

(v) $M_{ii} = \dfrac{\partial Q_i}{\partial \delta_i} = P_i - G_{ii}|V_i|^2$

Lecture 23

Decoupled Load Flow

In the NR method, the inverse of the Jacobian has to be computed at every iteration. When solving large interconnected systems, alternative solution methods are possible, taking into account certain observations are made of practical systems. These are given by

- Change in voltage magnitude $|V_i|$ at a bus primarily affects the flow of reactive power Q in the lines and leaves the real power P unchanged. This observation implies that $\frac{\partial Q_i}{\partial |V_j|}$ is much larger than $\frac{\partial P_i}{\partial |V_j|}$. Hence, in the Jacobian, the elements of the sub-matrix $[N]$, which contains terms that are partial derivatives of real power with respect to voltage magnitudes can be made zero.

- Change in voltage phase angle at a bus, primarily affects the real power flow P over the lines and the flow of Q is relatively unchanged. This observation implies that $\frac{\partial P_i}{\partial \delta_j}$ is much larger than $\frac{\partial Q_i}{\partial \delta_j}$. Hence, in the Jacobian the elements of the sub-matrix $[M]$, which contains terms that are partial derivatives of reactive power with respect to voltage phase angles can be made zero.

These observations reduce the NRLF linearised form of equation to

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} H & 0 \\ 0 & L \end{bmatrix} \begin{bmatrix} \Delta\delta \\ \dfrac{\Delta|V|}{|V|} \end{bmatrix} \tag{37}$$

From (37) it is obvious that the voltage angle corrections $\Delta\delta$ are obtained using real power residues ΔP and the voltage magnitude corrections $|\Delta V|$ are obtained from reactive power residues ΔQ. This equation can be solved through two alternate strategies as under:

Strategy-1

(i) Calculate $\Delta P^{(r)}, \Delta Q^{(r)}$ and $J^{(r)}$

(ii) Compute $\begin{bmatrix} \Delta\delta^{(r)} \\ \dfrac{\Delta|V^{(r)}|}{|V^{(r)}|} \end{bmatrix} = \left[J^{(r)}\right]^{-1} \begin{bmatrix} \Delta P^{(r)} \\ \Delta Q^{(r)} \end{bmatrix}$

(iii) Update δ and $|V|$.

(iv) Go to step (i) and iterate till convergence is reached.

Strategy-2

(i) Compute $\Delta P^{(r)}$ and Sub-matrix $H^{(r)}$. From (37) find $\Delta\delta^{(r)} = \left[H^{(r)}\right]^{-1}\Delta P^{(r)}$

(ii) Up date δ using $\delta^{(r+1)} = \delta^{(r)} + \Delta\delta^{(r)}$.

(iii) Use $\delta^{(r+1)}$ to calculate $\Delta Q^{(r)}$ and $L^{(r)}$

(iv) Compute $\dfrac{\Delta\left|V^{(r)}\right|}{\left|V^{(r)}\right|} = \left[L^{(r)}\right]^{-1}\Delta Q^{(r)}$

(v)Update, $\left|V^{(r+1)}\right| = \left|V^{(r)}\right| + \left|\Delta V^{(r)}\right|$

(vi) Go to step (i) and iterate till convergence is reached.

In the first strategy, the variables are solved simultaneously. In the second strategy the iteration is conducted by first solving for $\Delta\delta$ and using updated values of δ to calculate $\Delta|V|$. Hence, the second strategy results in faster convergence, compared to the first strategy.

MCQ’s

What is the element of the graph that is not included in the tree called?
a. Links
b. Branches
c. Oriented graph
d. All of these

What is an oriented graph?
a. A connection of network topology, represented by replacing all physical elements by lines.
b. A graph in which the direction is assigned to each branch.
c. A graph where at least one path exists between any two nodes of the graph.
d. None of these

Lecture 24

Fast Decoupled Load Flow

If the coefficient matrices are constant, the need to update the Jacobian at every iteration is eliminated. This has resulted in development of fast decoupled load Flow (FDLF). Here, certain assumptions are made based on the observations of practical power systems as under:

- $B_{ij} >> G_{ij}$ (Since the X/R ratio of transmission lines is high in well designed systems)

- The voltage angle difference $(\delta_i - \delta_j)$ between two buses in the system is very small. This means $\cos(\delta_i - \delta_j) \cong 1$ and $\sin(\delta_i - \delta_j) = 0.0$
- $Q_i << B_{ii}|V_i|^2$

With these assumptions the elements of the Jacobian become

$$H_{ik} = L_{ik} = -|V_i||V_k|B_{ik} \qquad (i \neq k)$$

$$H_{ii} = L_{ii} = -B_{ii}|V_i|^2$$

The matrix (37) reduces to

$$[\Delta P] = \left[|V_i||V_j|B'_{ij}\right][\Delta\delta]$$

$$[\Delta Q] = \left[|V_i||V_j|B''_{ij}\right]\left[\frac{\Delta|V|}{|V|}\right] \qquad (38)$$

Where B'_{ij} and B''_{ij} are negative of the susceptances of respective elements of the bus admittance matrix. In (38) if we divide LHS and RHS by $|V_i|$ and assume $|V_j| \cong 1$, we get,

$$\left[\frac{\Delta P}{|V|}\right] = \left[B'_{ij}\right]\left[\Delta \delta\right]$$

$$\left[\frac{\Delta Q}{|V|}\right] = \left[B''_{ij}\right]\left[\frac{\Delta |V|}{|V|}\right] \qquad (39)$$

Equations (39) constitute the Fast Decoupled load flow equations. Further simplification is possible by:

- Omitting effect of phase shifting transformers
- Setting off-nominal turns ratio of transformers to 1.0
- In forming B'_{ij}, omitting the effect of shunt reactors and capacitors which mainly affect reactive power
- Ignoring series resistance of lines in forming the Y_{bus}.

MCQ's

1. Certain graph has 4 nodes and 7 elements, the number of links is:

a) 3
b) 4
c) 11
d) 28

2. The number of nodes and the number of branches in a tree are related by:

a) b = n
b) b = n+1
c) b = n-1
d) b = 2n

Lecture 25

Comparison of Load Flow Methods

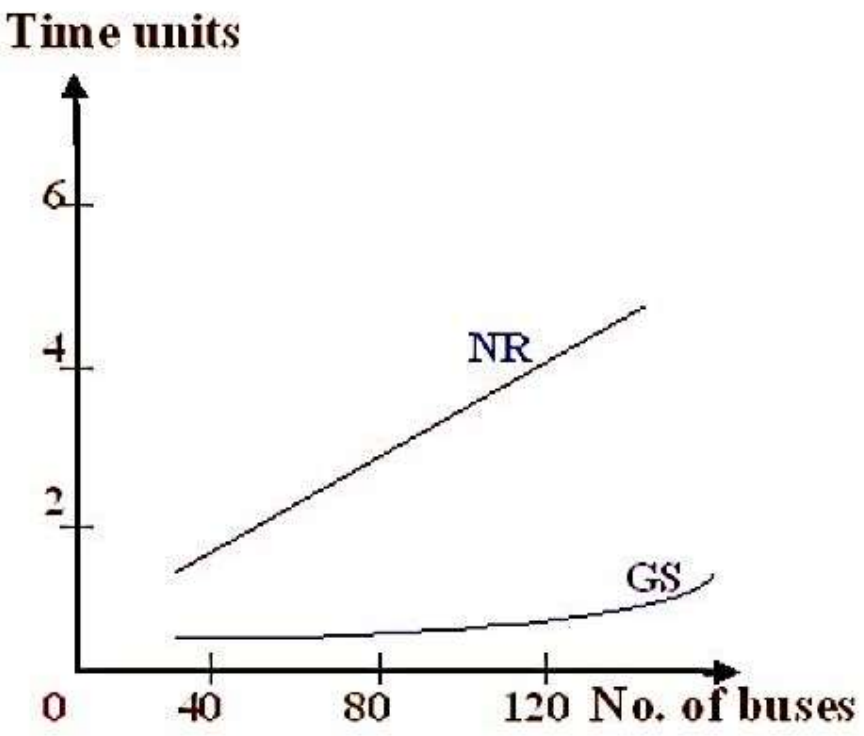


Figure 4. Time per Iteration in GS and NR methods

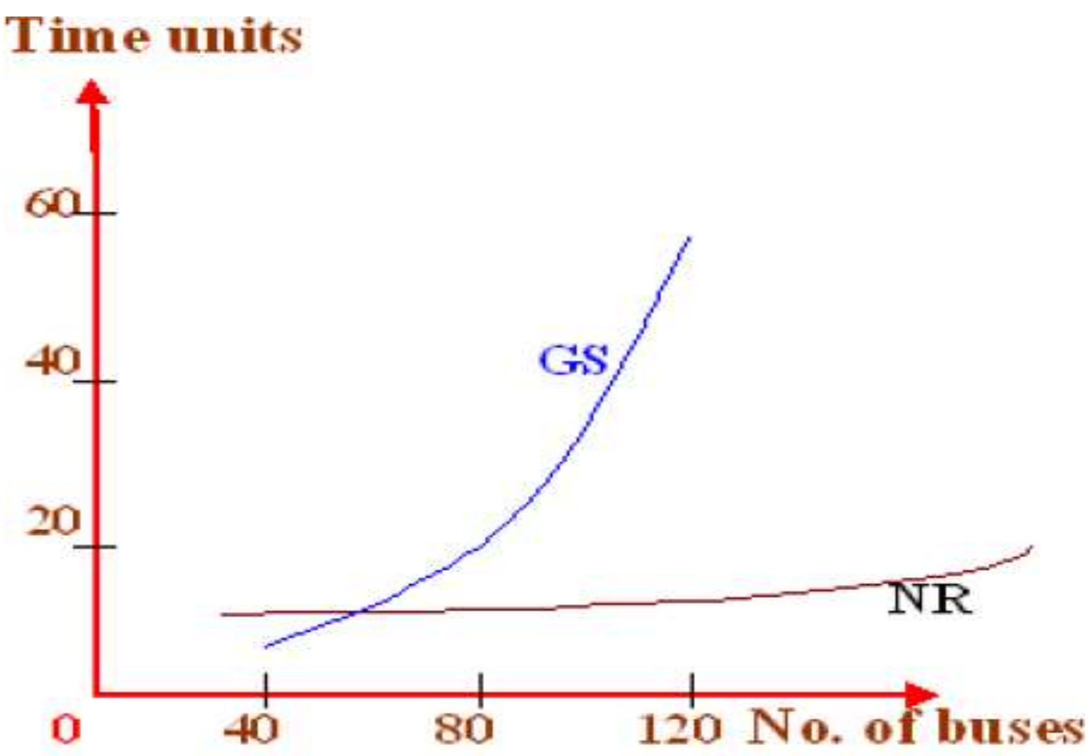


Figure 5. Total time of Iteration in GS and NR methods

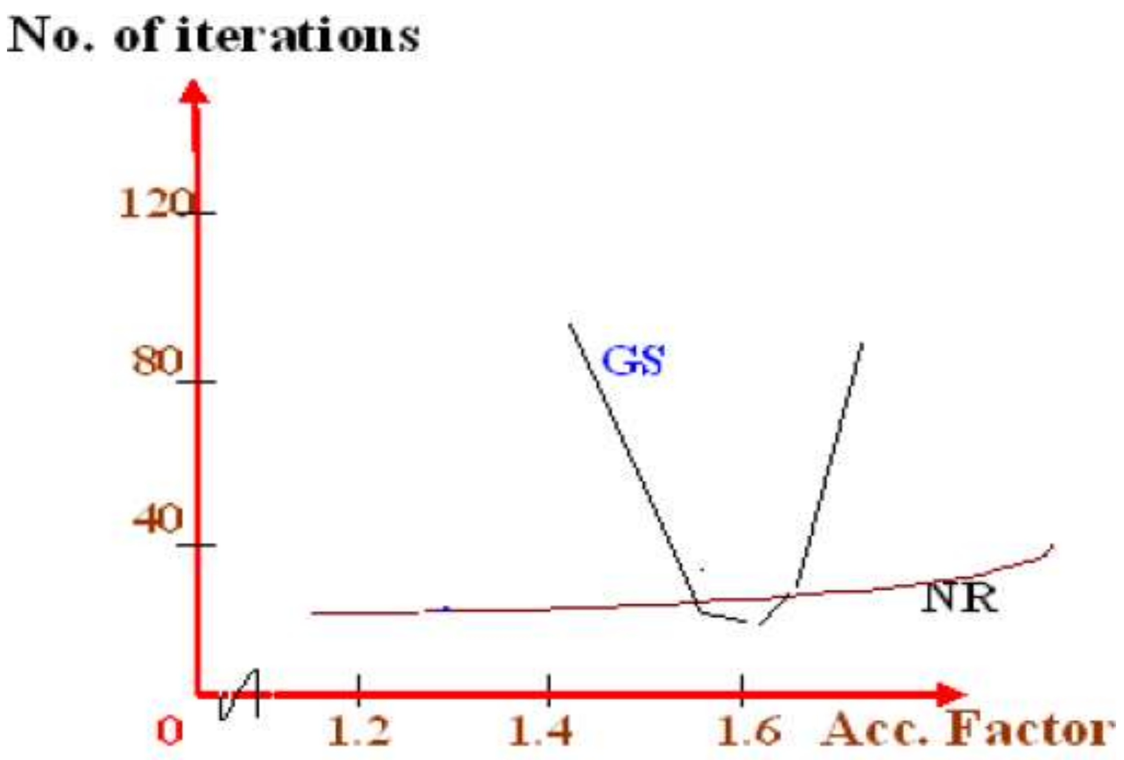


Figure 6. Influence of acceleration factor on load flow methods

Lecture 26

Solving Numericals

On NR Method

Lecture 27

Solving Numericals

On NR Method

Lecture 28

Solving Numericals

On NR Method

Lecture 29

Solving Numericals

On NR Method

Lecture 30

Solving Numericals

On NR Method

Thank You

Sri Adichunchanagiri Shikshana Trust (R)

SJB Institute of Technology

(Affiliated to Visvesvaraya Technological University, Belagavi& Approved by AICTE, New Delhi.)

Department of Electrical & Electronics Engineering

Power System Analysis – 2 [18EE71]

Module-4

Economic Operation of Power System & Unit Commitment

By:

Mr. Kubera U

Assistant Professor

EEE Department

SJB Institute of Technology

Lecture 31

Syllabus

Module 4:

Economic Operation of Power System: Introduction and Performance curves Economic generation scheduling neglecting losses and generator limits Economic generation scheduling including generator limits and neglecting losses Economic dispatch including transmission losses Derivation of transmission loss formula. Illustrative examples.

Unit Commitment: Introduction, Constraints and unit commitment solution by prior list method and dynamic forward DP approach (Flow chart and Algorithm only).

Introduction

- One of the earliest applications of on-line centralized control was to provide a central facility, to operate economically, several generating plants supplying the loads of the system.

- Modern integrated systems have different types of generating plants, such as coal fired thermal plants, hydel plants, nuclear plants, oil and natural gas units etc. The capital investment, operation and maintenance costs are different for different types of plants. The operation economics can again be subdivided into two parts,

Introduction Continued..

- ***Problem of economic dispatch***, which deals with determining the power output of each plant to meet the specified load, such that the overall fuel cost is minimized.
- ***Problem of optimal power flow***, which deals with minimum – loss delivery, where in the power flow, is optimized to minimize losses in the system. In this chapter we consider the problem of economic dispatch.

MCQ’s

1. The power system analysis is essential for

a)Planning the operation

b)Improvement and expansion of power system

c)Both a and b

d)None of the above

Lecture 32

Performance Curves

1. Input-Output Curve

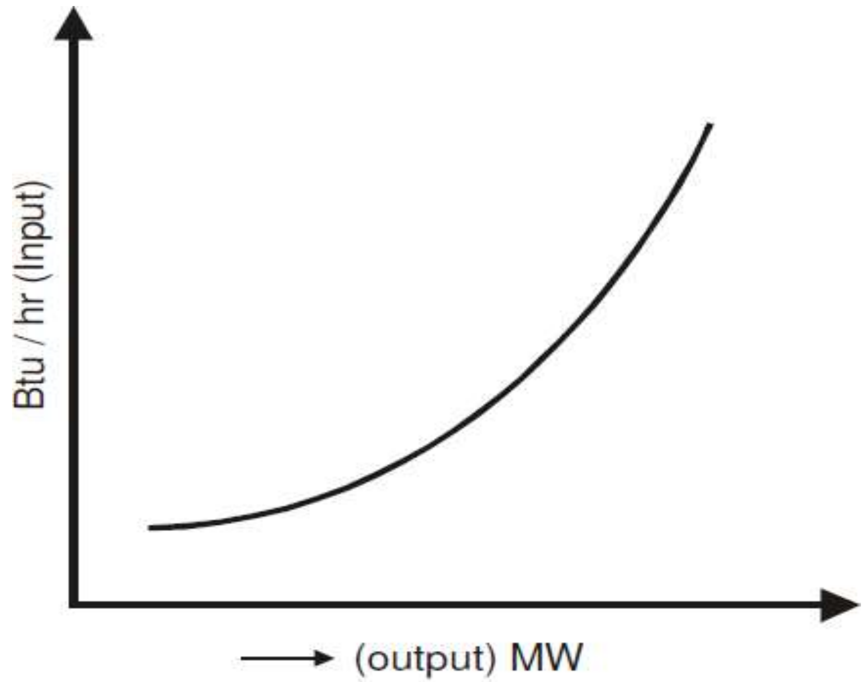
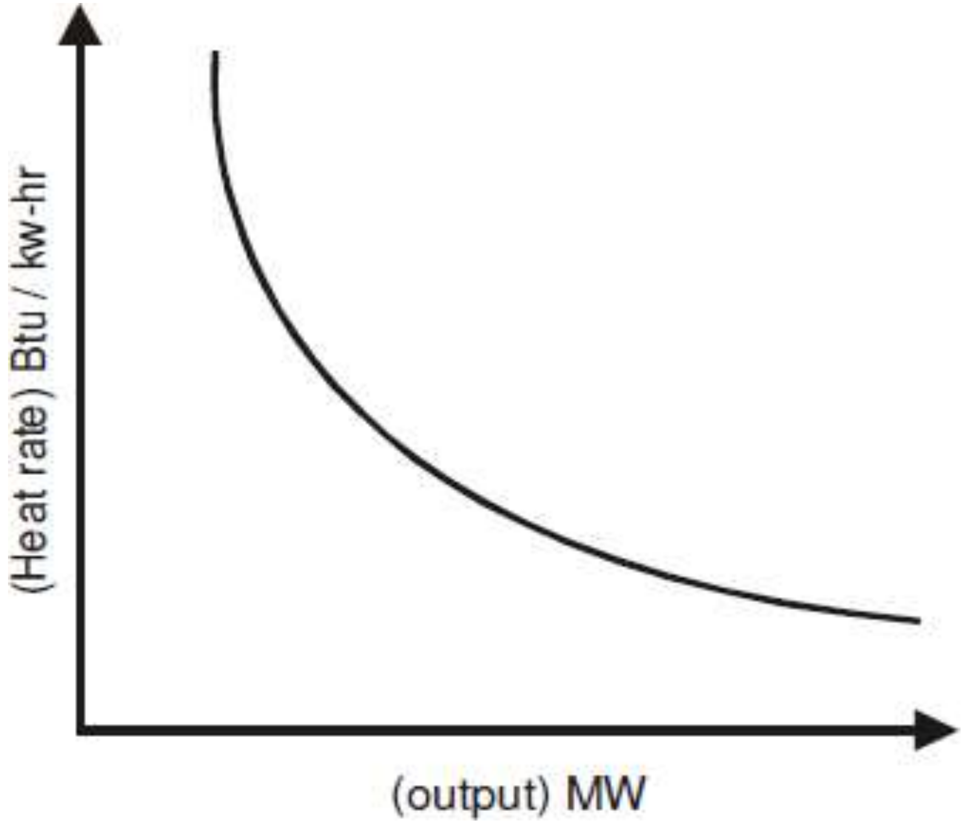


Fig 1: Input – output curve

2. Heat Rate Curve



3. Incremental Fuel Rate Curve

$$\text{Incremental fuel rate} = \frac{\Delta Input}{\Delta Output}$$

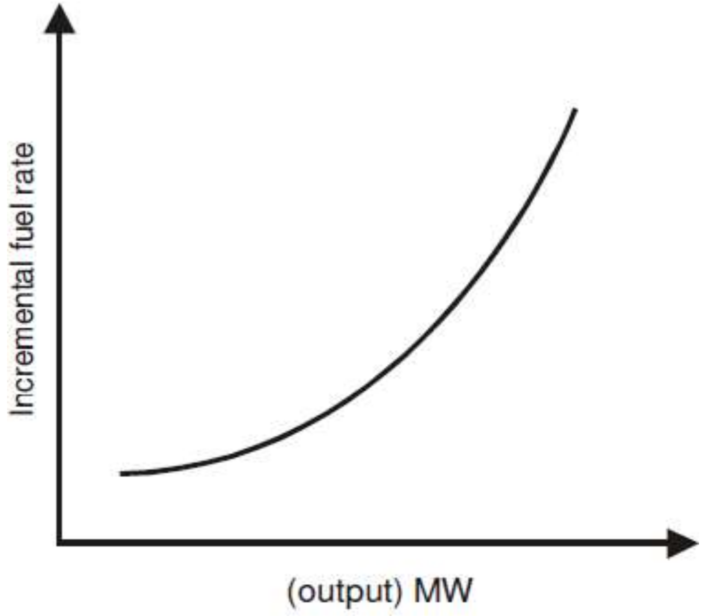


Fig 3: Incremental fuel rate curve

4. Incremental cost curve

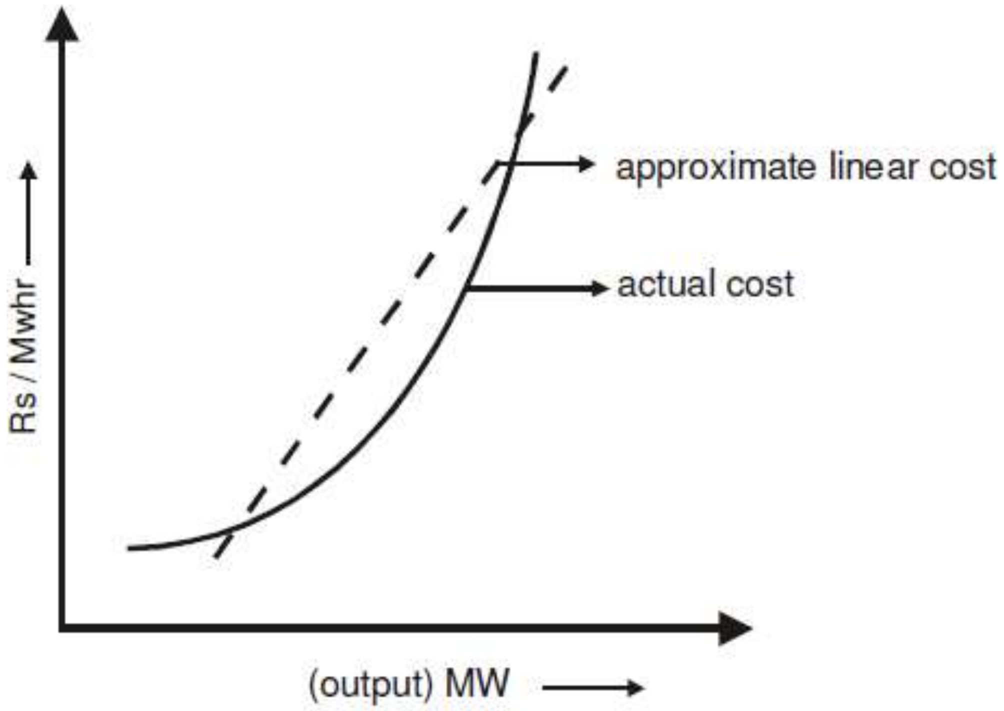


Fig 4: Incremental cost curve

$$F_i = a_i + b_i P_{Gi} + c_i P_{Gi}^2 \quad Rs / h$$

$$\frac{dF_i}{dP_{Gi}} = b_i + 2c_i P_{Gi} \qquad Rs / MWh$$

MCQ's

What is the element of the graph that is not included in the tree called?

a. Links

b. Branches

c. Oriented graph

d. All of these

What is an oriented graph?

a. A connection of network topology, represented by replacing all physical elements by lines.

b. A graph in which the direction is assigned to each branch.

c. A graph where at least one path exists between any two nodes of the graph.

d. None of these

Lecture 33

Economic Generation Scheduling Neglecting Losses And Generator Limits

The simplest case of economic dispatch is the case when transmission losses are neglected. The model does not consider the system configuration or line impedances. Since losses are neglected, the total generation is equal to the total demand P_D

Minimize $$F_T = \sum_{i=1}^{n_g} F_i$$

Such that $$\sum_{i=1}^{n_g} P_{Gi} = P_D$$

where
F_T = total cost.
P_{Gi} = generation of plant i.
P_D = total demand.

Lagrange Method for Solution of Economic Schedule

Minimize $$F_T = \sum_{i=1}^{n_g} F_i$$

Such that $$P_D = \sum_{i=1}^{n_g} P_{Gi} = 0$$

The augmented cost function is given by

$$\pounds = \mathrm{F_T} + \lambda\left(P_D - \sum_{i=1}^{n_g} P_{Gi} \right)$$

The minimum is obtained when

$$\frac{\partial \pounds}{\partial P_{Gi}} = 0 \ \text{ and } \ \frac{\partial \pounds}{\partial \lambda} = 0$$

$$\frac{\partial \pounds}{\partial \mathrm{P_{Gi}}} = \frac{\partial F_T}{\partial P_{Gi}} - \lambda = 0$$

$$\frac{\partial \pounds}{\partial \lambda} = P_D - \sum_{i=1}^{n_g} P_{Gi} = 0$$

$$\frac{\partial F_T}{\partial P_{Gi}} = \frac{\partial F_i}{\partial P_{Gi}} = \frac{dF_i}{dP_{Gi}}$$

Using the above,

$$\frac{\partial F_i}{\partial P_{Gi}} = \frac{dF_i}{dP_{Gi}} = \lambda \; ; \qquad i = 1 \ldots\ldots n_g$$

We can write

$$b_i + 2c_i P_{Gi} = \lambda \qquad i = 1 \ldots\ldots n_g$$

$$P_{Gi} = \frac{\lambda - b_i}{2c_i}$$

We know in a loss less system

$$\sum_{i=1}^{n_g} P_{Gi} = P_D$$

Substituting (8.16) we get

$$\sum_{i=1}^{n_g} \frac{\lambda - b_i}{2c_i} = P_D$$

An analytical solution of λ is obtained from (8.17) as

$$\lambda = \frac{P_D + \sum_{i=1}^{n_g} \frac{b_i}{2c_i}}{\sum_{i=1}^{n_g} \frac{1}{2c_i}}$$

Numericals

Lecture 34

Economic Schedule Including Limits on Generator (Neglecting Losses)

$$\sum_{i=1}^{n_g} P_{Gi} = P_D$$

and the inequality constraint

$$P_{Gi\,(min)} \leq P_{Gi} \leq P_{Gi\,(max)}\,;\, i = 1, \ldots\ldots\ldots n_g$$

Numericals

MCQ’s

1. Certain graph has 4 nodes and 7 elements, the number of links is:
a) 3
b) 4
c) 11
d) 28

2. The number of nodes and the number of branches in a tree are related by:
a) $b = n$
b) $b = n+1$
c) $b = n-1$
d) $b = 2n$

Economic Dispatch Including Transmission Losses

Minimize $$F_T = \sum_{i=1}^{n_g} F_i$$

Such That $$\sum_{i=1}^{n_g} P_{Gi} = P_D + P_L$$

where P_L is the total loss.

The Lagrange function is now written as

$$£ = F_T - \lambda\left(\sum_{i=1}^{n_g} P_{Gi} - P_D - P_L\right) = 0$$

The minimum point is obtained when

$$\frac{\partial £}{\partial \mathrm{P_{Gi}}} = \frac{\partial F_T}{\partial P_{Gi}} - \lambda\left(1 - \frac{\partial P_{\mathrm{L}}}{\partial \mathrm{P_{Gi}}}\right) = 0\,;\ \ \mathrm{i} = 1\ldots\ldots \mathrm{n_g}$$

$$\frac{\partial £}{\partial \lambda} = \sum_{i=1}^{n_g} P_{Gi} - P_D + P_L = 0 \qquad \text{(Same as the constraint)}$$

Since $\dfrac{\partial F_T}{\partial P_{Gi}} = \dfrac{dF_i}{dP_{Gi}}$, (8.27) can be written as

$$\frac{dF_i}{dP_{Gi}} + \lambda\frac{\partial P_{\mathrm{L}}}{\partial \mathrm{P_{Gi}}} = \lambda$$

$$\lambda = \frac{dF_i}{dP_{Gi}}\left(\frac{1}{1-\frac{\partial P_L}{\partial P_{Gi}}}\right)$$

The term $\frac{1}{1-\frac{\partial P_L}{\partial P_{Gi}}}$ is called the penalty factor of plant i, $\mathbf{L_i}$. The coordination equations including losses are given by

$$\lambda = \frac{dF_i}{dP_{Gi}} L_i ; i = 1 \ldots\ldots n_g$$

$$P_L = \sum_m \sum_n P_{Gm} \; B_{mn} \; P_{Gn} + \sum_n P_{Gn} \, B_{no} + B_{oo}$$

$$P_L = \sum_m \sum_n P_{Gm} \; B_{mn} \; P_{Gn}$$

$$P_L = B_{11} \, P_{G1} + 2 \, B_{12} \, P_{G1} \, P_{G2} + B_{22} \, P_{G2}^{\;2}$$

Numericals

MCQ's

1. The dimension of bus incidence matrix is:

a. e x n

b. e x (n-1)

c. e x (n+1)

d. e x (n+2)

2. How many fundamental cutsets will be generated for a graph with 'n' number of nodes?

a. n+1

b. n-1

c. n^2(n-1)

d. n/ n-1

Lecture 35

Derivation of Transmission Loss Formula

An accurate method of obtaining general loss coefficients has been presented by Kron. The method is elaborate and a simpler approach is possible by making the following assumptions:

- All load currents have same phase angle with respect to a common reference
- The ratio X / R is the same for all the network branches.

Consider the simple case of two generating plants connected to an arbitrary number of loads through a transmission network as shown in Fig below.

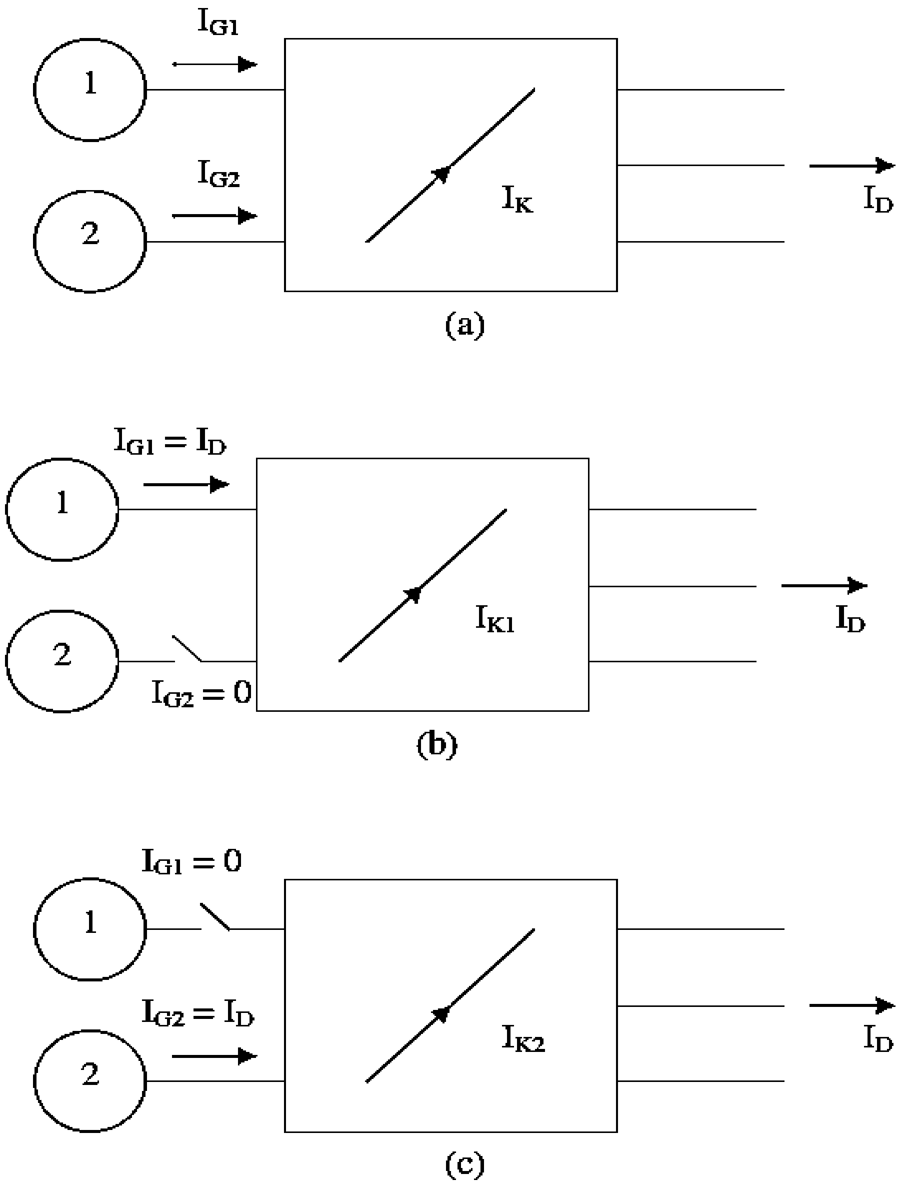


Fig Two plants connected to a number of loads through a transmission network

Let's assume that the total load is supplied by only generator 1 as shown in Fig 8.9b. Let the current through a branch K in the network be I_{K1}. We define

$$N_{K1} = \frac{I_{K1}}{I_D}$$

It is to be noted that $I_{G1} = I_D$ in this case. Similarly with only plant 2 supplying the load current I_D, as shown in Fig 8.9c, we define

$$N_{K2} = \frac{I_{K2}}{I_D}$$

N_{K1} and N_{K2} are called current distribution factors and their values depend on the impedances of the lines and the network connection. They are independent of I_D. When both generators are supplying the load, then by principle of superposition

$$I_K = N_{K1}\, I_{G1} + N_{K2}\, I_{G2}$$

where I_{G1}, I_{G2} are the currents supplied by plants 1 and 2 respectively, to meet the demand I_D. Because of the assumptions made, I_{K1} and I_D have same phase angle, as do I_{K2} and I_D. Therefore, the current distribution factors are real rather than complex. Let

$$I_{G1} = |I_{G1}|\angle\sigma_1 \text{ and } I_{G2} = |I_{G2}|\angle\sigma_2 .$$

$$|I_K|^2 = \left(N_{K1}|I_{G1}|\cos\sigma_1 + N_{K2}|I_{G2}|\cos\sigma_2\right)^2 + \left(N_{K1}|I_{G1}|\sin\sigma_1 + N_{K2}|I_{G2}|\sin\sigma_2\right)^2$$

$$= \begin{array}{l} N_{K1}{}^2|I_{G1}|^2\left[\cos^2\sigma_1 + \sin^2\sigma_1\right] + N_{K2}{}^2|I_{G2}|^2\left[\cos^2\sigma_2 + \sin^2\sigma_2\right] \\ + 2\left[N_{K1}|I_{G1}|\cos\sigma_1 N_{K2}|I_{G2}|\cos\sigma_2 + N_{K1}|I_{G1}|\sin\sigma_1 N_{K2}|I_{G2}|\sin\sigma_2\right] \end{array}$$

$$= N_{K1}{}^2|I_{G1}|^2 + N_{K2}{}^2|I_{G2}|^2 + 2N_{K1}N_{K2}|I_{G1}||I_{G2}|\cos(\sigma_1 - \sigma_2)$$

Now $|I_{G1}| = \dfrac{P_{G1}}{\sqrt{3}|V_1|\cos\phi_1}$ and $|I_{G2}| = \dfrac{P_{G2}}{\sqrt{3}|V_2|\cos\phi_2}$

where P_{G1}, P_{G2} are three phase real power outputs of plant1 and plant 2; V_1, V_2 are the line to line bus voltages of the plants and ϕ_1, ϕ_2 are the power factor angles.

The total transmission loss in the system is given by

$$P_L = \sum_K 3|I_K|^2 R_K$$

where the summation is taken over all branches of the network and R_K is the branch resistance. Substituting we get

$$P_L = \frac{P_{G1}{}^2}{|V_1|^2(\cos\phi_1)^2}\sum_K N_{K1}{}^2 R_K + \frac{2P_{G1}P_{G2}\cos(\sigma_1-\sigma_2)}{|V_1||V_2|\cos\phi_1\cos\phi_2}\sum_K N_{K1}N_{K2}R_K$$

$$+\frac{P_{G2}{}^2}{|V_2|^2(\cos\phi_2)^2}\sum_K N_{K2}{}^2 R_K$$

$$P_L = P_{G1}{}^2 B_{11} + 2P_{G1}P_{G2}B_{12} + P_{G2}{}^2 B_{22}$$

where $$B_{11} = \frac{1}{|V_1|^2(\cos\phi_1)^2}\sum_K N_{K1}{}^2 R_K$$

$$B_{12} = \frac{\cos(\sigma_1 - \sigma_2)}{|V_1||V_2|\cos\phi_1 \cos\phi_2} \sum_K N_{K1} N_{K2} R_K$$

$$B_{22} = \frac{1}{|V_2|^2 (\cos\phi_2)^2} \sum_K N_{K2}{}^2 R_K$$

The loss – coefficients are called the B – coefficients and have unit MW^{-1}.

For a general system with n plants the transmission loss is expressed as

$$P_L = \frac{P_{G1}{}^2}{|V_1|^2 (\cos\phi_1)^2} \sum_K N_{K1}{}^2 + \ldots\ldots + \frac{P_{Gn}{}^2}{|V_n|^2 (\cos\phi_n)^2} \sum_K N_{Kn}{}^2 R_K$$

$$+ 2 \sum_{\substack{p,q=1 \\ p \neq q}}^{n} \frac{P_{GP} P_{Gq} \cos(\sigma_p - \sigma_q)}{|V_p||V_q| \cos\phi_p \cos\phi_q} \sum_k N_{KP} N_{Kq} R_K$$

In a compact form

$$P_L = \sum_{p=1}^{n}\sum_{q=1}^{n} P_{Gp} B_{Pq} P_{Gq}$$

$$B_{Pq} = \frac{\cos(\sigma_p - \sigma_q)}{|V_p||V_q|\cos\phi_p \cos\phi_q} \sum_K N_{KP} N_{Kq} R_K$$

B – Coefficients can be treated as constants over the load cycle by computing them at average operating conditions, without significant loss of accuracy.

Numericals

MCQ's

What is the simplified diagram called, after omitting all resistances, static loads, capacitance of the transmission lines and magnetizing circuit of the transformer?

a. Single line diagram

b. Resistance diagram

c. Reactance diagram

d. Both (a) and (b)

e. None of these

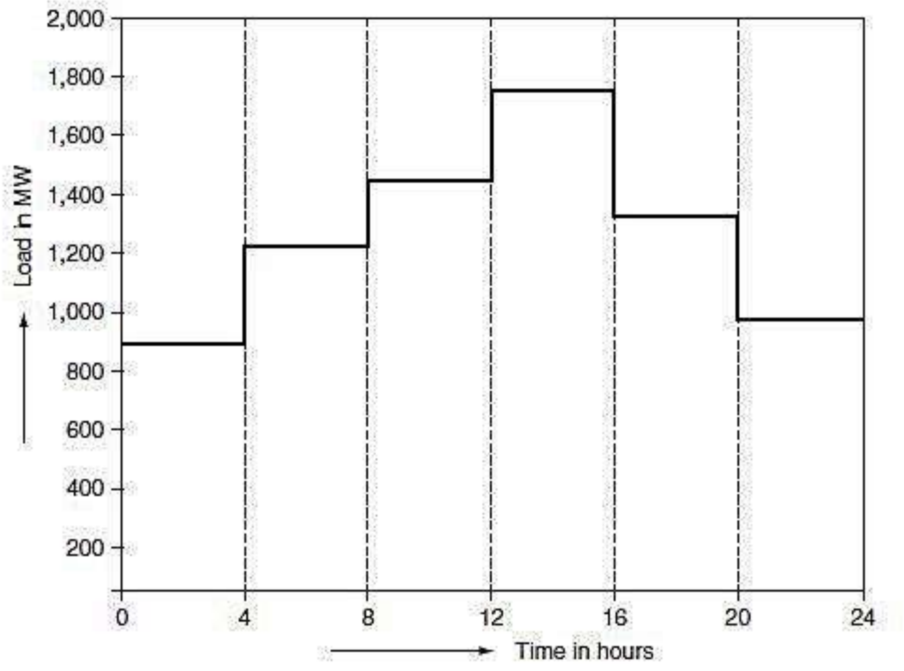
Lecture 36

Numericals

Lecture 37

Unit Commitment

The total load of the power system is not constant but varies throughout the day and reaches a different peak value from one day to another. It follows a particular hourly load cycle over a day. There will be different discrete load levels at each period as shown in Figure



Due to the above reason, it is not advisable to run all available units all the time, and it is necessary to decide in advance which generators are to startup, when to connect them to the network, the sequence in which the operating units should be shut down, and for how long. The computational procedure for making such decisions is called **unit commitment** (UC), and a unit when scheduled for connection to the system is said to be committed.

Lecture 38

Constraints in Unit Commitment

There are many constraints to be considered in solving the UC problem.

1.Spinning reserve

$$\text{Spinning reserve} = \begin{bmatrix} \text{Total generation output of all} \\ \text{synchronized units at a} \\ \text{particular time} \end{bmatrix} - \begin{bmatrix} \text{Load at that time} + \\ \text{Losses at that time} \end{bmatrix}$$

$$\therefore\ P_{G_{sp}} = \sum_{i=1}^{n} P_{G_i} - (P_D + P_L)$$

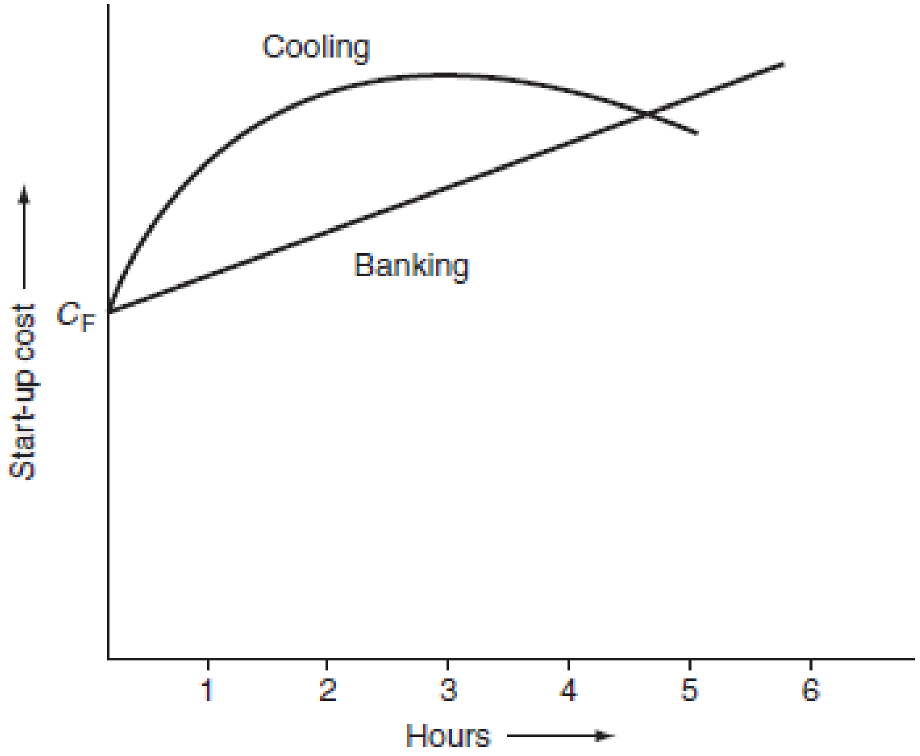
2. Thermal unit constraints

Minimum up-time: During the minimum up-time, once the unit is operating (up state), it should not be turned off immediately.

Minimum down-time: The minimum down-time is the minimum time during which the unit is in ‘down’ state, i.e., once the unit is decommitted, there is a minimum time before it can be recommitted.

Crew constraints: If a plant consists of two or more units, they cannot both be turned on at the same time since there are not enough crew members to attend to both units while starting up.

Start-up cost**:** In addition to the above constraints, because the temperature and the pressure of the thermal unit must be moved slowly, a certain amount of energy must be expended to bring the unit on-line and is brought into the UC problem as a start-up cost.



maintaining unit at operating temperature (MBtu/hr), and t the time the unit was cooled (hr).

Start-up cost when cooling = Cc (1 – e-t/α) C + CF;

Start-up cost when banking =Ct × t × C + CF.

3. Hydro-constraints

As pointed out already that the UC problem is of much importance for the scheduling of thermal units, it is not the meaning of UC that cannot be completely separated from the scheduling of a hydro-unit. The hydro-thermal scheduling will be explained as separated from the UC problem. Operation of a system having both hydro and thermal plants is, however, far more complex as hydro-plants have negligible operation costs, but are required to operate under constraints of water available for hydro-generation in a given period of time.

The problem of minimizing the operating cost of a hydro-thermal system can be viewed as one of minimizing the fuel cost of thermal plants under the constraint of water availability for hydro-generation over a given period of operation.

4. Must run

It is necessary to give a must-run reorganization to some units of the plant during certain events of the year, by which we yield the voltage support on the transmission network or for such purpose as supply of steam for uses outside the steam plant itself.

5. Fuel constraints

A system in which some units have limited fuel or else have constraints that require them to burn a specified amount of fuel in a given time presents a most challenging UC problem.

Lecture 39

Unit Commitment - Solution Methods

The most important techniques for the solution of a UC problem are:

i. Priority-list method.

ii. Dynamic programming (DP) method.

iii. Lagrange’s relaxation (LR) method.

Now, the priority-list method and the DP method are discussed here.

1. Priority list method

It is the simplest unit commitment solution which consists of creating a priority list of units.

Full load average production cost= Net heat rate at full load X Fuel

Cost Assumptions:

1. No load cost is zero
2. Unit input-output characteristics are linear between zero output and full load
3. Startup costs are a fixed amount
4. Ignore minimum up time and minimum down time

Steps to be followed

1. Determine the full load average production cost for each units
2. Form priority order based on average production cost
3. Commit number of units corresponding to the priority order
4. Calculate P_{G1}, P_{G2} ,P_{GN} from economic dispatch problem for the feasible combinations only.
5. For the load curve shown, Assume load is dropping or decreasing, determine whether dropping the next unit will supply generation & spinning reserve. If not, continue as it is If yes, go to the next step.

6. Determine the number of hours H, before the unit will be needed again.

7. Check H< minimum shut down time. If not, go to the last step If yes, go to the next step.

8. Calculate two costs, one is the Sum of hourly production for the next H hours with the unit up and second one is the Recalculate the same for the unit down + startup cost for either cooling or banking.

9. Repeat the procedure until the priority.

Merits:

1. No need to go for N combinations
2. Take only one constraint
3. Ignore the minimum up time & down time
4. Complication reduced

Demerits:

1. Startup cost are fixed amount
2. No load costs are not considered

Lecture 40

2. Dynamic-Programming Solution:

Dynamic programming has many advantages over the enumeration scheme, the chief advantage being a reduction in the dimensionality of the problem. Suppose we have found units in a system and any combination of them could serve the (single) load.

There would be a maximum of 24 - 1 = 23 combinations to test. However, if a strict priority order is imposed, there are only four combinations to try:

- Priority 1 unit
- Priority 1 unit + Priority 2 unit
- Priority 1 unit + Priority 2 unit + Priority 3 unit
- Priority 1 unit + Priority 2 unit + Priority 3 unit + Priority 4 unit

The imposition of a priority list arranged in order of the full-load average cost rate would result in a theoretically correct dispatch and commitment only if:

1. No load costs are zero.
2. Unit input-output characteristics are linear between zero output and full load.
3. There are no other restrictions.
4. Start-up costs are a fixed amount.

In the dynamic-programming approach that follows, we assume that:

1. A state consists of an array of units with specified units operating and
2. The start-up cost of a unit is independent of the time it has been off-line
3. There are no costs for shutting down a unit.
4. There is a strict priority order, and in each interval a specified minimum the rest off-line. (i.e., it is a fixed amount).amount of capacity must be operating.

Forward DP Approach:

The recursive algorithm to compute the minimum cost in hour K with combination I is

$F_{cost}(K,I) = \min[P_{cost}(K,I) + S_{cost}(K-1,L:K,I) + F_{cost}(K-1,L)]$

$F_{cost}(K, I)$ = least total cost to arrive at state (K , I)

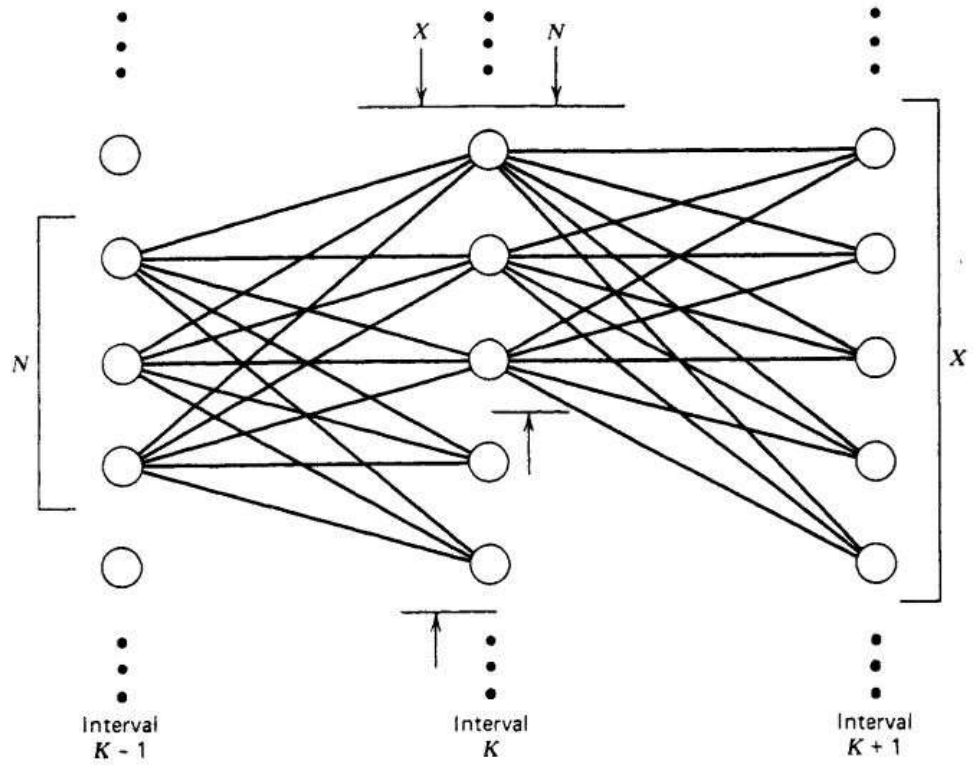
$P_{cost}(KI,)$ = production cost for state(K ,I)

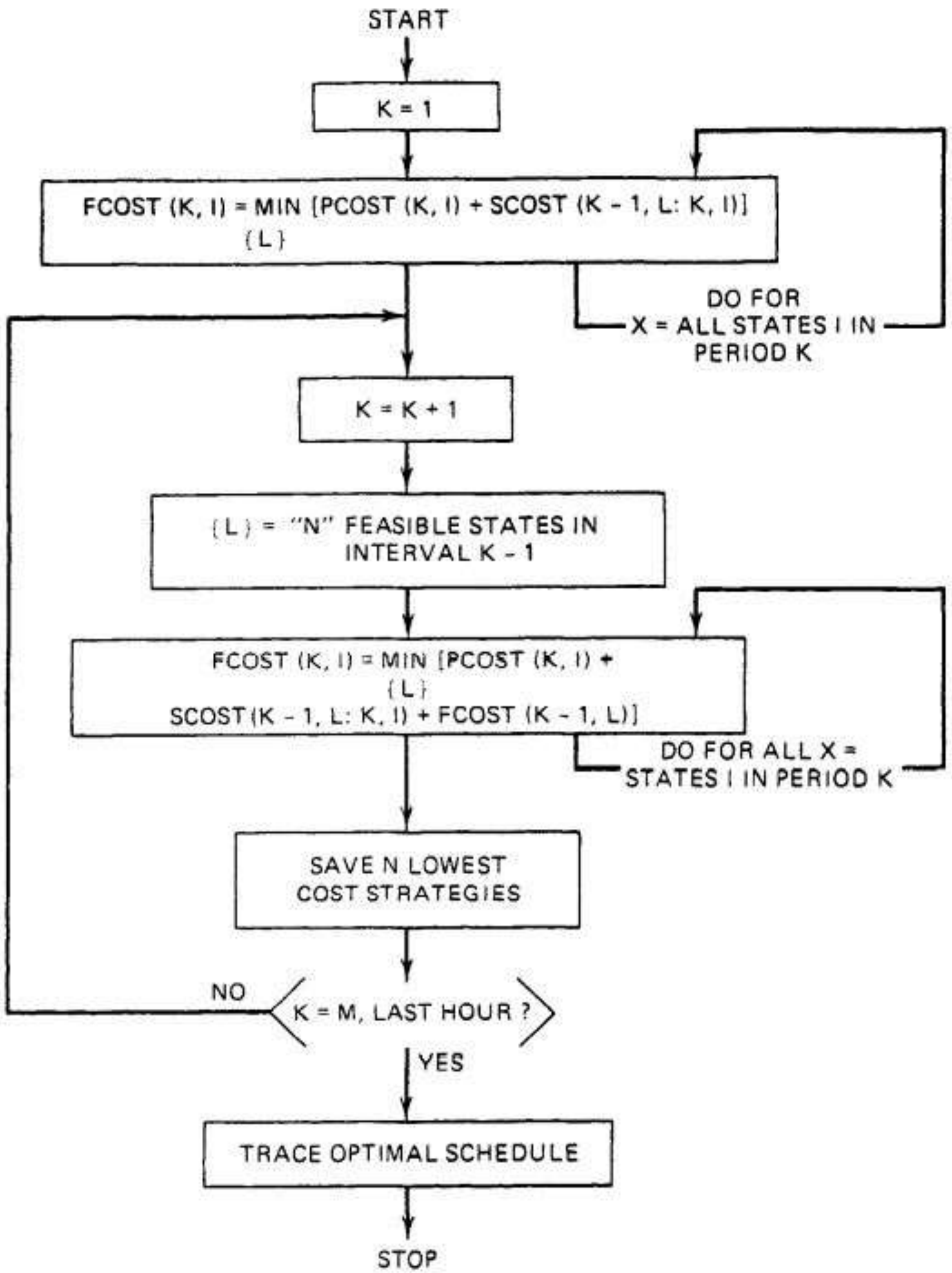
$S_{cost}(K - 1, L: K , I)$= transition cost from state (K - 1, L) to state(K , I)

Note that two new variables, X and N, have been introduced

X = number of states to search each period

N = number of strategies, or paths, to save at each step.




START
K = 1
FCOST (K, I) = MIN [PCOST (K, I) + SCOST (K - 1, L: K, I)]
{L}
DO FOR
X = ALL STATES I IN
PERIOD K
K = K + 1
{L} = "N" FEASIBLE STATES IN
INTERVAL K - 1
FCOST (K, I) = MIN [PCOST (K, I) +
{L}
SCOST (K - 1, L: K, I) + FCOST (K - 1, L)]
DO FOR ALL X =
STATES I IN PERIOD K
SAVE N LOWEST
COST STRATEGIES
NO
K = M, LAST HOUR ?
YES
TRACE OPTIMAL SCHEDULE
STOP

Thank You

Sri Adichunchanagiri Shikshana Trust (R)

SJB Institute of Technology

(Affiliated to Visvesvaraya Technological University, Belagavi& Approved by AICTE, New Delhi.)

Department of Electrical & Electronics Engineering

Power System Analysis – 2 [18EE71]

Module-4

Symmetrical Fault Analysis & Power System Stability

By:

Mr. Kubera U

Assistant Professor

EEE Department

SJB Institute of Technology

Lecture 41

Syllabus

Module 5:

Symmetrical Fault Analysis: Z Bus Formulation by Step by step building algorithm without mutual coupling between the elements by addition of link and addition of branch. Illustrative examples. Z bus Algorithm for Short Circuit Studies excluding numerical.T1

Power System Stability: Numerical Solution of Swing Equation by Point by Point method and Runge Kutta Method. Illustrative examples.

Introduction

- Short circuit study is one of the basic power system analysis problems. It is also known as fault analysis. When a fault occurs in a power system, bus voltages reduces and large current flows in the lines. This may cause damage to the equipments. Hence faulty section should be isolated from the rest of the network immediately on the occurrence of a fault. This can be achieved by providing relays and circuit breakers.

- The calculation of currents in network elements for different types of faults occurring at different locations is called **SHORT CIRCUIT STUDY**. The results obtained from the short circuit study are used to find the relay settings and the circuit breaker ratings which are essential for power system protection.

MCQ’s

1. The power system analysis is essential for

a)Planning the operation

b)Improvement and expansion of power system

c)Both a and b

d)None of the above

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Z Bus Formulation by Step by Step Building Algorithm

The bus impedance matrix is the inverse of the bus admittance matrix. An alternative method is possible, based on an algorithm to form the bus impedance matrix directly from system parameters and the coded bus numbers. The bus impedance matrix is formed adding one element at a time to a partial network of the given system. The performance equation of the network in bus frame of reference in impedance form using the currents as independent variables is given in matrix form by

$$\overline{E}_{bus} = [Z_{bus}]\overline{I}_{bus}$$

When expanded so as to refer to a *n* bus system, (9) will be of the form

$$E_1 = Z_{11}I_1 + Z_{12}I_2 + \ldots\ldots + Z_{1k}I_k \ldots + Z_{1n}I_n$$

$$\vdots$$

$$\vdots$$

$$E_k = Z_{k1}I_1 + Z_{k2}I_2 + \ldots\ldots + Z_{kk}I_k + \ldots + Z_{kn}I_n$$

$$\vdots$$

$$\vdots$$

$$E_n = Z_{n1}I_1 + Z_{n2}I_2 + \ldots\ldots + Z_{nk}I_k + \ldots + Z_{nn}I_n \qquad (10)$$

Now assume that the bus impedance matrix Zbus is known for a partial network of *m* buses and a known reference bus. Thus, Zbus of the partial network is of dimension *m*x*m*. If now a new element is added between buses *p* and *q* we have the following two possibilities:

- *p* is an existing bus in the partial network and *q* is a new bus; in this case *p*-*q* is a **branch** added to the p-network as shown in Fig 1a, and

- both *p* and *q* are buses existing in the partial network; in this case *p*-*q* is a **link** added to the p-network as shown in Fig 1b

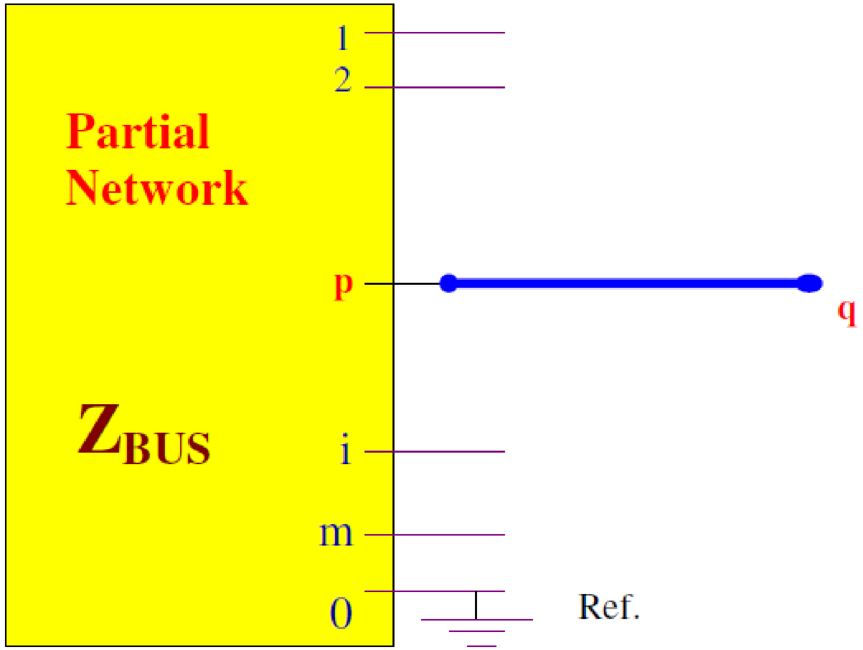

Fig 1a. Addition of branch p-q

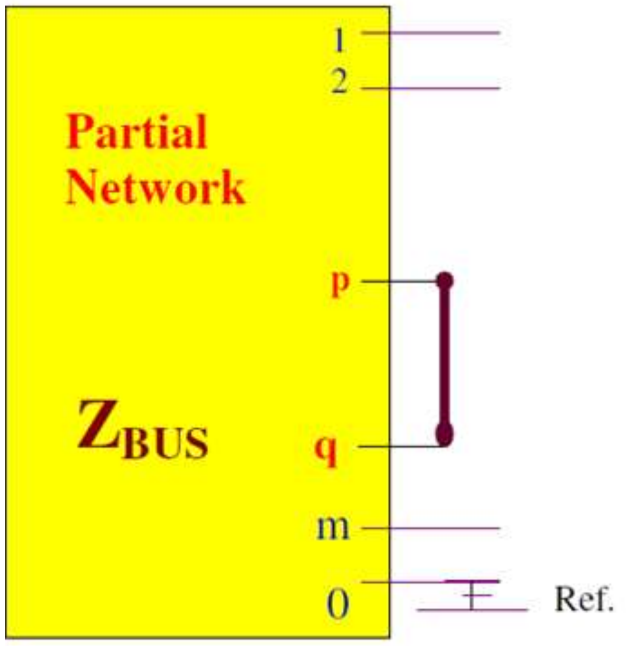

Fig 1b. Addition of link p-q

MCQ's

What is the element of the graph that is not included in the tree called?

a. Links

b. Branches

c. Oriented graph

d. All of these

What is an oriented graph?

a. A connection of network topology, represented by replacing all physical elements by lines.

b. A graph in which the direction is assigned to each branch.

c. A graph where at least one path exists between any two nodes of the graph.

d. None of these

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Addition of a Branch

$$\begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_p \\ \vdots \\ E_m \\ E_q \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1p} & \cdots & Z_{1m} & Z_{1q} \\ Z_{21} & Z_{22} & \cdots & Z_{2p} & \cdots & Z_{2m} & Z_{2q} \\ \vdots & & & & & & \\ Z_{p1} & Z_{p2} & \cdots & Z_{pp} & \cdots & Z_{pm} & Z_{pq} \\ \vdots & & & & & & \\ Z_{m1} & Z_{m2} & \cdots & Z_{mp} & \cdots & Z_{mm} & Z_{mq} \\ Z_{q1} & Z_{q2} & \cdots & Z_{qp} & \cdots & Z_{qm} & Z_{qq} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_p \\ \vdots \\ I_m \\ I_q \end{bmatrix} \qquad (11)$$

To find Zqi:

The elements of last row-q and last column-q are determined by injecting a current of 1.0 pu at the bus-i and measuring the voltage of the bus-q with respect to the reference bus-0, as shown in Fig.2. Since all other bus currents are zero, we have from (11) that

Ek = Zki Ii = Zki " k = 1, 2,…i p,….m, q (13)

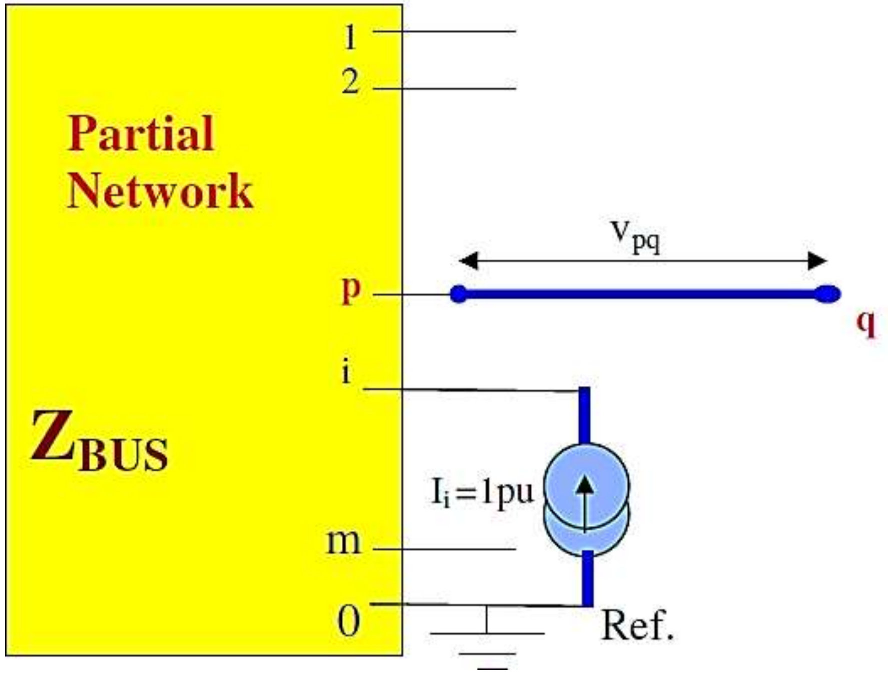
Hence, Eq = Zqi ; Ep = Zpi ………

Also, Eq=Ep -vpq ; so that Zqi = Zpi - vpq " i =1, 2,…i…....p,….m, _q (14)

To find vpq:

In terms of the primitive admittances and voltages across the elements, the current through the elements is given by

$$\begin{bmatrix} i_{pq} \\ \bar{i}_{rs} \end{bmatrix} = \begin{bmatrix} y_{pq,pq} & \bar{y}_{pq,rs} \\ \bar{y}_{rs,pq} & \bar{y}_{rs,rs} \end{bmatrix} \begin{bmatrix} v_{pq} \\ \bar{v}_{rs} \end{bmatrix} \qquad (15)$$



where i_{pq} is current through element *p-q*

$\bar{i}_{rs}$ is vector of currents through elements of the partial network

v_{pq} is voltage across element *p-q*

$y_{pq,pq}$ is self – admittance of the added element

$\bar{y}_{pq,rs}$ is the vector of mutual admittances between the added elements *p-q* and elements *r-s* of the partial network.

$\bar{v}_{rs}$ is vector of voltage across elements of partial network.

$\bar{y}_{rs,pq}$ is transpose of $\bar{y}_{pq,rs}$.

$\bar{y}_{rs,rs}$ is the primitive admittance of partial network.

Since the current in the added branch p-q, is zero, $i_{pq} = 0$. We thus have from (15),

$$i_{pq} = y_{pq,pq} v_{pq} + \bar{y}_{pq,rs} \bar{v}_{rs} = 0 \qquad (16)$$

Solving, $v_{pq} = -\dfrac{\bar{y}_{pq,rs}\bar{v}_{rs}}{y_{pq,pq}}$ or

$$v_{pq} = -\frac{\bar{y}_{pq,rs}\left(\bar{E}_r - \bar{E}_s\right)}{y_{pq,pq}} \qquad (17)$$

Using (13) and (17) in (14), we get

$$Z_{qi} = Z_{pi} + \frac{\bar{y}_{pq,rs}\left(\bar{Z}_{ri} - \bar{Z}_{si}\right)}{y_{pq,pq}} \quad i = 1,2.....m; i \neq q \qquad (18)$$

To find zqq:

The element Z_{qq} can be computed by injecting a current of 1pu at bus-q, $I_q = 1.0$ pu.

As before, we have the relations as under:

$E_k = Z_{kq}\, I_q = Z_{kq}$ $\quad \forall$ k = 1, 2,…i…....p,….m, q (19)

Hence, $E_q = Z_{qq}$; $E_p = Z_{pq}$; Also, $E_q = E_p - v_{pq}$; so that $Z_{qq} = Z_{pq} - v_{pq}$ (20)

Since now the current in the added element is $i_{pq} = -I_q = -1.0$, we have from (15)

$$i_{pq} = y_{pq,pq} v_{pq} + \bar{y}_{pq,rs}\bar{v}_{rs} = -1$$

Solving, $v_{pq} = -1 + \dfrac{\bar{y}_{pq,rs}\bar{v}_{rs}}{y_{pq,pq}}$

$$v_{pq} = -1 + \frac{\bar{y}_{pq,rs}\left(\bar{E}_r - \bar{E}_s\right)}{y_{pq,pq}} \tag{21}$$

Using (19) and (21) in (20), we get

$$Z_{qq} = Z_{pq} + \frac{1 + \bar{y}_{pq,rs}\left(\bar{Z}_{rq} - \bar{Z}_{sq}\right)}{y_{pq,pq}} \tag{22}$$

Special Cases

The following special cases of analysis concerning ZBUS building can be considered with respect to the addition of branch to a p-network

Case (a): If there is no mutual coupling then elements of $\bar{y}_{pq,rs}$ are zero. Further, if p is the reference node, then E_p=0. thus,

$$Z_{pi} = 0 \qquad i = 1,2.....m : i \neq q$$

And $\quad Z_{pq} = 0.$

Hence, from (18) (22) $\quad Z_{qi} = 0 \qquad i = 1,2......m; i \neq q$

And $\quad Z_{qq} = z_{pq,pq} \qquad \backslash \quad (23)$

Case (b): If there is no mutual coupling and if p is not the ref. bus, then, from (18) and (22), we again have,

$$Z_{qi} = Z_{pi}, \quad i = 1,2....m; i \neq q$$

$$Z_{qq} = Z_{pq} + z_{pq,pq} \qquad (24)$$

Addition of a Link

$$\begin{bmatrix} E_1 \\ E_2 \\ \vdots \\ E_p \\ \vdots \\ E_m \\ E_l \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & \cdots & Z_{1p} & \cdots & Z_{1m} & Z_{1q} \\ Z_{21} & Z_{22} & \cdots & Z_{2p} & \cdots & Z_{2m} & Z_{2q} \\ \vdots & & & & & & \\ Z_{p1} & Z_{p2} & \cdots & Z_{pp} & \cdots & Z_{pm} & Z_{pq} \\ \vdots & & & & & & \\ Z_{m1} & Z_{m2} & \cdots & Z_{mp} & \cdots & Z_{mm} & Z_{mq} \\ Z_{l1} & Z_{l2} & \cdots & Z_{li} & \cdots & Z_{lm} & Z_{ll} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_p \\ \vdots \\ I_m \\ I_l \end{bmatrix} \qquad (25)$$

To find Zli:

The elements of last row-l and last column-l are determined by injecting a current of 1.0 pu at the bus-i and measuring the voltage of the bus-q with respect to the reference bus-0, as shown in Fig.3. Further, the current in the added element is made zero by connecting a voltage source, el in series with element p-q, as shown. Since all other bus currents are zero, we have from (25) that

$$E_k = Z_{ki} I_i = Z_{ki} \qquad \forall\ k = 1, 2, \ldots i \ldots\ldots p, \ldots m, l \qquad (27)$$

Hence, $e_l = E_l = Z_{li}$; $E_p = Z_{pi}$; $E_p = Z_{pi}$ ....

Also, $e_l = E_p - E_q - v_{pq}$;

So that $Z_{li} = Z_{pi} - Z_{qi} - v_{pq} \quad \forall\ i=1,2,\ldots i \ldots p, \ldots q, \ldots m, \neq l \qquad (28)$

To find Vpq:

In terms of the primitive admittances and voltages across the elements, the current through the elements is given by

$$\begin{bmatrix} i_{pl} \\ \bar{i}_{rs} \end{bmatrix} = \begin{bmatrix} y_{pl,pl} & \bar{y}_{pl,rs} \\ \bar{y}_{rs,pl} & \bar{y}_{rs,rs} \end{bmatrix} \begin{bmatrix} v_{pl} \\ \bar{v}_{rs} \end{bmatrix} \qquad (29)$$

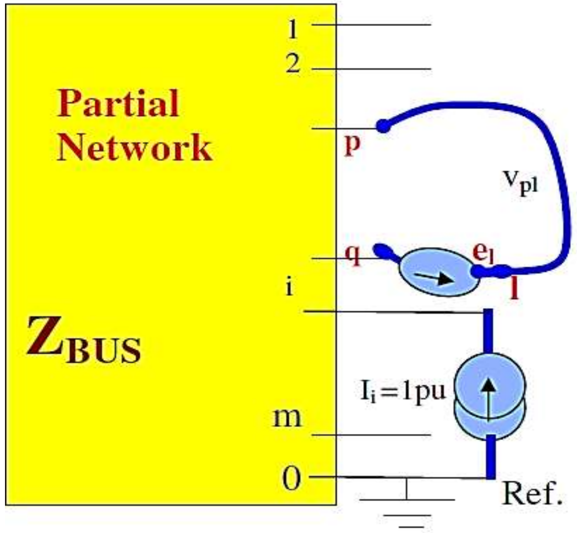


Fig.3 Calculation for Z_{li}

where i_{pl} is current through element *p-q*

$\bar{i}_{rs}$ is vector of currents through elements of the partial network

v_{pl} is voltage across element *p-q*

$y_{pl,pl}$ is self – admittance of the added element

$\bar{y}_{pl,rs}$ is the vector of mutual admittances between the added elements *p-q* and elements *r-s* of the partial network.

$\bar{v}_{rs}$ is vector of voltage across elements of partial network.

$\bar{y}_{rs,pl}$ is transpose of $\bar{y}_{pl,rs}$.

$\bar{y}_{rs,rs}$ is the primitive admittance of partial network.

Since the current in the added branch p-l, is zero, $i_{pl} = 0$. We thus have from (29),

$$i_{pl} = y_{pl,pl} v_{pl} + \bar{y}_{pl,rs} \bar{v}_{rs} = 0 \tag{30}$$

Solving, $v_{pl} = -\dfrac{\bar{y}_{pl,rs} \bar{v}_{rs}}{y_{pl,pl}}$ or

$$v_{pl} = -\frac{\bar{y}_{pl,rs}\left(\bar{E}_r - \bar{E}_s\right)}{y_{pl,pl}} \tag{31}$$

However,

$$\bar{y}_{pl,rs} = \bar{y}_{pq,rs}$$

And $$y_{pl,pl} = y_{pq,pq} \tag{32}$$

Using (27), (31) and (32) in (28), we get

$$Z_{li} = Z_{pi} - Z_{qi} + \frac{\bar{y}_{pq,rs}\left(\bar{Z}_{ri} - \bar{Z}_{si}\right)}{y_{pq,pq}} \qquad i = 1,2......m; i \neq l \tag{33}$$

To find Z_{ll}:

The element Z_{ll} can be computed by injecting a current of 1pu at bus-l, $I_l = 1.0$ pu. As before, we have the relations as under:

$E_k = Z_{kl} I_l = Z_{kl} \qquad \forall\ k = 1, 2, \ldots i \ldots\ldots p, \ldots q, \ldots . m, l \qquad (34)$

Hence, $e_l = E_l = Z_{ll}$; $E_p = Z_{pl}$;

Also, $e_l = E_p - E_q - v_{pl}$;

$$Z_{ll} = Z_{pl} - Z_{ql} + \frac{1 + \bar{y}_{pq,rs}\left(\bar{Z}_{rl} - \bar{Z}_{sl}\right)}{y_{pq,pq}} \qquad (38)$$

Special Cases Contd….

The following special cases of analysis concerning Z_{BUS} building can be considered with respect to the addition of link to a p-network.

Case (c): If there is no mutual coupling, then elements of $\bar{y}_{pq,rs}$ are zero. Further, if *p* is the reference node, then $E_p=0$. thus,

$$Z_{li} = -Z_{qi}, \quad i = 1,2 \ldots m; i \neq l$$

$$Z_{ll} = -Z_{ql} + z_{pq,pq} \qquad (39)$$

Lecture 44

Numericals Z_{bus} Building Method

Lecture 45

Numericals Z_{bus} Building Method

Lecture 46

Numerical Solution of Swing Equation by Point by Point method

There are several sophisticated methods for solving the swing equation. The step-by-step or point-by-point method is conventional, approximate but well tried and proven method. This method determines the changes in the rotor angular position during a short interval of time.

Consider the swing equation:

$$M\frac{d^2\delta}{dt^2} = P_S - P_{max}\sin\delta = P_A$$

The solution $\delta(t)$ is obtained at discrete intervals of time with interval spread of Δt uniform throughout.

Accelerating power, PA and change in speed, which are continuous function of time and are described as below,

1. The accelerating power PA computed at the beginning of an interval is assumed to remain constant from the middle of the preceding interval to the middle of the interval being considered, as illustrated in Fig.1.
2. The angular rotor velocity ω', i.e., $d\delta/dt$ (over and above synchronous velocity $\omega 0$) is assumed to remain constant throughout any interval at the value computed for the middle of the interval, as illustrated in Fig.1.

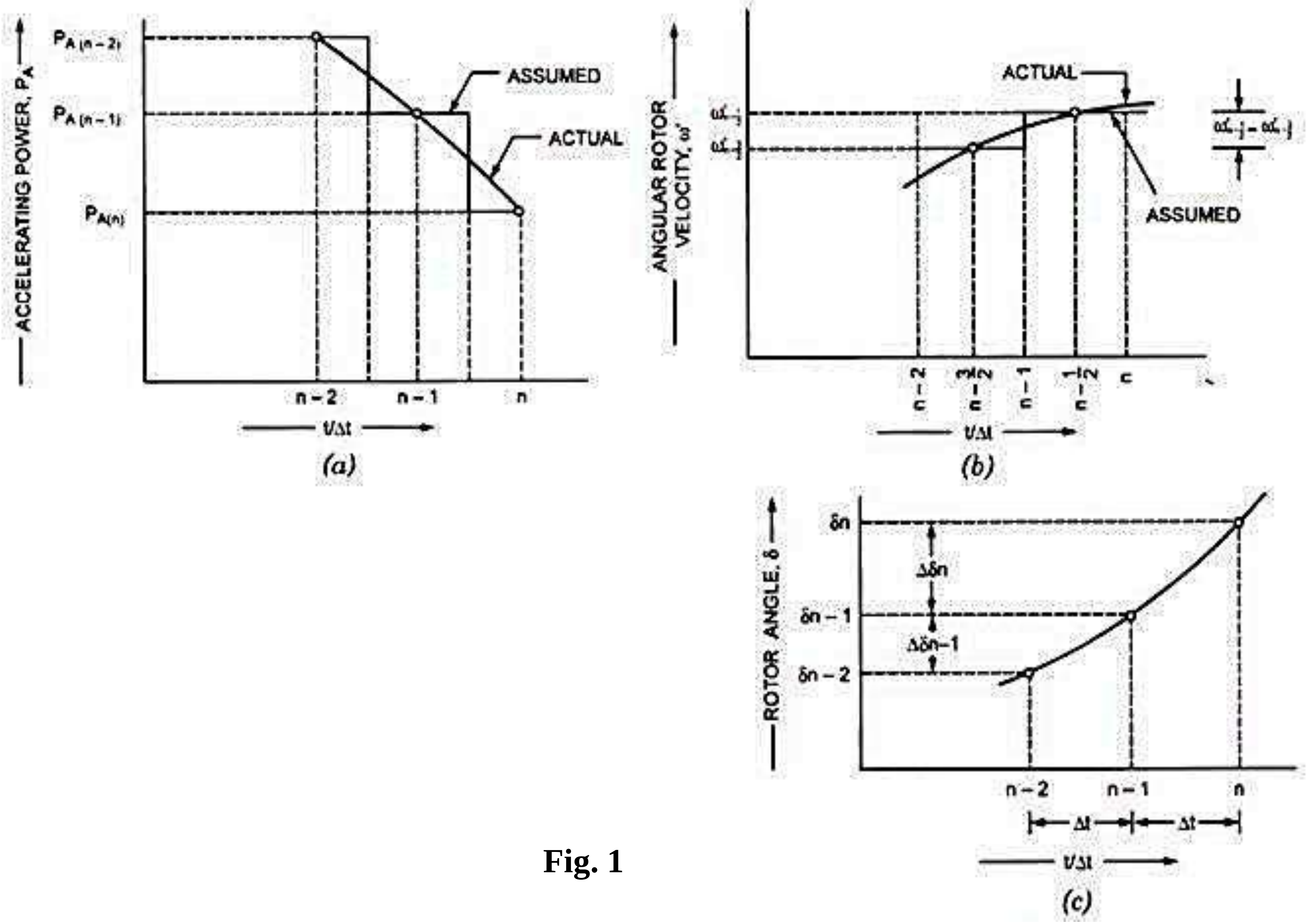

ACCELERATING POWER, P_A
$P_{A(n-2)}$
$P_{A(n-1)}$
$P_{A(n)}$
ASSUMED
ACTUAL
n − 2
n − 1
n
t/Δt
(a)
ANGULAR ROTOR VELOCITY, ω'
ACTUAL
ASSUMED
t/Δt
(b)
ROTOR ANGLE, δ
δn
Δδn
δn − 1
Δδn−1
δn − 2
n − 2
n − 1
n
Δt
Δt
t/Δt
(c)

Fig. 1

In Fig.1 the numbering on t/Δt axis pertains to the end of intervals.

The equation for accelerating power at the end of the $(n-1)^{th}$ interval or for nth interval can be written as

$$P_{A(n-1)} = P_S - P_{max} \sin \delta_{n-1}$$

where δ_{n-1} has been earlier calculated.

The change in velocity caused due to $P_{A(n-1)}$ assumed to remain constant over Δt from (n – 3/2) to (n – 1/2),

$$\Delta\omega'_{n-1/2} = \omega'_{n-1/2} - \omega'_{n-3/2}$$

$$= \frac{\Delta t}{M} P_{A(n-1)}$$

The change in rotor angle δ during $(n-1)$th interval.

$$\Delta\delta_{n-1} = \delta_{n-1} - \delta_{n-2} = \Delta t \omega'_{n-\frac{3}{2}}$$

and during the nth interval, $\Delta\delta_n = \delta_n - \delta_{n-1} = \Delta t \omega'_{n-\frac{1}{2}}$

Subtracting Eq. (7.55) from Eq. (7.56) we have

$$\Delta\delta_n - \Delta\delta_{n-1} = \Delta t\left(\omega'_{n-\frac{1}{2}} - \omega'_{n-\frac{3}{2}}\right) = \frac{(\Delta t)^2}{M} P_{A(n-1)}$$

$\because$ from Eq. (7)

$$\omega'_{n-\frac{1}{2}} - \omega'_{n-\frac{3}{2}} = \frac{(\Delta t)}{M} P_{A(n-1)}$$

$$\text{or} \quad \Delta\delta_n = \Delta\delta_{n-1} + \frac{P_{A(n-1)}}{M}(\Delta t)^2$$

$$\therefore \delta_n = \delta_{n-1} + \Delta\delta_n$$

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Runge-Kutta Method

In Range - Kutta method, the changes in dependent variables are calculated from a given set of formulae, derived by using an approximation, to replace a truncated Taylor's series expansion. The formulae for the Runge - Kutta fourth order approximation, for solution of two simultaneous differential equations are given below;

$$\text{Given } \frac{dx}{dt} = f_x(x, y, t)$$

$$\frac{dy}{dt} = f_y(x, y, t)$$

Starting from initial values x_0, y_0, t_0 and step size h, the updated values are

$$x_1 = x_0 + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)$$

$$y_1 = y_0 + \frac{1}{6}(l_1 + 2l_2 + 2l_3 + l_4)$$

where $k_1 = f_x(x_0, y_0, t_0)\, h$

$$k_2 = f_x\left(x_0 + \frac{k_1}{2}, y_0 + \frac{l_1}{2}, t_0 + \frac{h}{2}\right) h$$

$$k_3 = f_x\left(x_0 + \frac{k_2}{2}, y_0 + \frac{l_2}{2}, t_0 + \frac{h}{2}\right) h$$

$k_4 = f_x(x_0 + k_3, y_0 + l_3, t_0 + h)\, h$

$l_1 = f_y(x_0, y_0, t_0)\, h$

$$l_2 = f_y\left(x_0 + \frac{k_1}{2}, y_0 + \frac{l_1}{2}, t_0 + \frac{h}{2}\right) h$$

$$l_3 = f_y\left(x_0 + \frac{k_2}{2}, y_0 + \frac{l_2}{2}, t_0 + \frac{h}{2}\right) h$$

$l_4 = f_y(x_0 + k_3, y_0 + l_3, t_0 + h)\, h$

The two first order differential equations to be solved to obtain solution for the swing equation are:

$$\frac{d\delta}{dt} = \omega$$

$$\frac{d\omega}{dt} = \frac{P_a}{M} = \frac{P_m - P_{max}\sin\delta}{M}$$

Starting from initial value δ_0, ω_0, t_0 and a step size of Δt the formulae are as follows

$$k_1 = \omega_0 \, \Delta t$$

$$l_1 = \left[\frac{P_m - P_{max}\sin\delta_0}{M}\right] \Delta t$$

$$k_2 = \left(\omega_0 + \frac{l_1}{2}\right) \Delta t$$

$$l_2 = \left[\frac{P_m - P_{max}\sin\left(\delta_0 + \frac{k_1}{2}\right)}{M}\right] \Delta t$$

$$k_3 = \left(\omega_0 + \frac{l_2}{2}\right) \Delta t$$

$$l_3 = \left[\frac{P_m - P_{max} \sin\left(\delta_0 + \frac{k_2}{2}\right)}{M} \right] \Delta t$$

$$k_4 = (\omega_0 + l_3)\ \Delta t$$

$$l_4 = \left[\frac{P_m - P_{max} \sin(\delta_0 + k_3)}{M} \right] \Delta t$$

$$\delta_1 = \delta_0 + \frac{1}{6}\ [k_1 + 2k_2 + 2k_3 + k_4]$$

$$\omega_1 = \omega_0 + \frac{1}{6}\ [l_1 + 2l_2 + 2l_3 + l_4]$$

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Numericals

Lecture 49

Numericals

Lecture 50

Numericals

Thank You